Instituto Tecnológico y de Estudios Superiores de Monterrey

Campus Monterrey

School of Engineering and Sciences

Phase II Lepage-type CUSUM charts for joint monitoring of location and scale

A thesis presented by

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Submitted to the
School of Engineering and Sciences
in partial fulfillment of the requirements for the degree of

Master of Science

In

Industrial Engineering

From the

Universitat Politècnica de Catalunya

Monterrey Nuevo León, July 11th, 2017

Instituto Tecnológico y de Estudios Superiores de Monterrey

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Declaration of Authorship

I, Víctor Aguilar Lleyda, declare that this thesis titled, “Phase II Lepage-type CUSUM charts for joint monitoring of location and scale” and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Víctor Aguilar Lleyda
Monterrey Nuevo León, July 11th, 2017

Dedication

Thanks for all your unconditional confidence, support, patience, and encouragement.

First, to my tutor Víctor Tercero, Ph.D., and professors Alvaro Cordero, Ph.D. and William Jay Conover, Ph.D., they were responsible for my grown interest in the field of Statistical Process Control.

To my father, Antonio, who advised me to enter the world of engineering, and to my mother, Margarida, and brother, David, who motivated me during harsh times.

You were my main motivation for pushing through this work.

Acknowledgements

I would like to express my deepest gratitude to all those who have been side by side with me.

Thank you Tecnológico de Monterrey for the support on tuition and excellent treatment during my stay here. It’s been a pleasure to be part of this organization.

And as a mobility student, to my hometown university, Universitat Politècnica de Catalunya (UPC) for providing my engineering education and for offering me the opportunity to study abroad.

Phase II Lepage-type CUSUM charts for joint monitoring of location and scale

by

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Abstract

Most control charts in nonparametric statistics tend to focus on detecting changes in the location or scale parameters. Since it is ideal to monitor both, researchers have proposed to combine them into a single statistic. From there, the Lepage-type family of tests was born, and it's been updated with many versions. Chowdhury, Mukherjee, and Chrakaborti proposed merging the Ansari-Bradley and Wilcoxon Rank Sum tests into a single plotting statistic able to track persistent changes in the location and scale parameters in Phase II in the form of a CUSUM chart, using a reference sample from Phase I. The work from Guerrero (2016) based on several tests proposed by Conover et al. (1981) shows that there may be many statistics able to perform better than the Ansari-Bradley for monitoring the scale parameter. In the present work, we propose three Control charts based on the mentioned recommendations. These are the LP-M, LP-FK, and LP-SR CUSUM charts, in which the Ansari-Bradley statistic is replaced for the Mood, the Fligner and Killeen, and the Squared ranks tests respectively. Via Monte Carlo simulations, Control limits are tabulated for practitioners, plus the In-control and Out-of-control performance of the charts are calculated and compared with the CUSUM-Lepage Chart proposed by Chowdhury. This will help to choose the most useful CUSUM chart for each scenario. An example using real data illustrates how the proposed Control Charts must be implemented.

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Chapter 1. Introduction

1.1 Motivation

After World War II, Japan became an incredibly crippled country. The occupying forces had to help rebuild communications to inform the people of Japan that the war was over and the United States was no longer an enemy. Engineer Homer Sarasohn was sent from the Massachusetts Institute of Technology to share American manufacturing methods with the Japanese leaders. They thought the use of statistics was the key on why the allies had won and wanted to know more about it.

Years later, in 1950, W. Edwards Deming traveled to Japan to teach quality methods, giving emphasis on quantifying variation and predicting future process performance. Originally, engineer Walter A. Shewhart was supposed to attend, however, since he was too ill to leave the US, Deming took his place. The reason why Shewhart was meant to go initially was that he created the first control chart while he was working for Bell Labs in the 1920's. So it was actually Deming who was influenced by Shewhart and later on he became the foremost advocate of Shewhart's work (Best & Neuhauser, 2006).

It was back in 1924 when Shewhart came with a file containing a diagram and a few lines explaining how it worked. That's when began what is called now Statistical Process Control (SPC), by creating the first Control Chart (also known as Shewhart Chart after 1931). Shewhart's work was focused on explaining how variability could be originated by natural causes (or aleatory) or special causes (or assignable).

When only common causes of variation exist in a production process, we can consider it to be stable, or in-control (IC). When in that production process the variability is high or systematic so that we are far from the specified requirements, and that variability can be directly related to some specific cause, the component responsible for that variation is the special cause. In this scenario, the process is considered unstable or out-of-control (OOC). The major goal of SPC is to distinguish special cause variation from common cause variation and give a signal as soon as special cause variation occurs (Qiu 2013).

It's been almost a century since that event, and nowadays companies recognize the importance of SPC for monitoring sequential processes to make sure they work stably and satisfactorily. The application of SPC can be divided into three main phases: understanding the process and its specification limits, eliminating the assignable sources of variation, and monitoring the ongoing production process to detect significant changes in mean or variance. The use of control charts is directly related to the late phase so that we can act as in the first two phases. Therefore, during the last decades, a generous amount of different control charts have been developed to detect and reduce variability.

The Shewhart chart was valuable since it is effortless to use and understand. However, it’s been shown by researchers that it’s only useful when there are significant changes or “shifts” in the process. What happens when changes are more subtle? Well, in 1954 Page came with a chart called Cumulative sum, or CUSUM chart. This technique was created to track processes getting Out-of-control when the changes were subtle and maintained over time. This is a phenomenon pretty common in the modern industry, so the CUSUM chart has been well received and has suffered many improvements over time. One of them is the integration of tests able to detect changes in a manufacturing process even when we have little information about how is data distributed. This new branch was called “nonparametric control charts”, since they are not based on parameterized families of probability distributions. Though this methodology behaves worse than techniques assuming normality when data is actually normal, they are way superior in detecting changes in the location or scale when the distribution of data is unknown.

Regarding this concept, many nonparametric tests have been highlighted to detect either the change happens in the mean, like the Wilcoxon Rank Sum (WRS) test, or whether it's going on in the process variance, like the Mood test (1954). Since checking two charts at the same time can be confusing for practitioners and the overall confidence of the control chart scheme is reduced by using more than one chart, some authors opted to combine them into one single chart that detects both changes in location or scale parameters.

The pioneer in this aspect was Lepage (1971), who combined the WRS and Ansari-Bradley statistics (1960) to create what is called Lepage-type test. His inheritance has been used with many other statistics and types of control charts.

In 2015, Chowdhury, Mukherjee and Chakraborti combined these methodologies in order to create a nonparametric chart that could monitor both mean and variance of a process while using a single plotting statistic, in a CUSUM chart.

The tests used in that chart were the inspiration for this study since the work by Guerrero (2016) showed that the Ansari-Bradley test can be substituted for a better one in terms of performance for detecting changes in the scale parameter, while keeping the other qualities that this type of chart offers.

1.2 Problem Statement and Context

This thesis deals with the problem of designing new Control Charts to monitor the Phase II location and scale parameters of processes, when the assumption that data follows a known distribution can't be made, by using a modification of the CUSUM-Lepage (CL) chart proposed by Chowdhury et al. (2015).

These proposed charts follow a Lepage approach, as done by Ross et al. (2011) and introduced by Lepage (1971) which involve the combination of the Wilcoxon Rank Sum test for location, and the Fligner and Killeen, Squared Ranks or Mood tests for the scale parameter, instead of the Ansari-Bradley test. The selection of the first two is based on the study by Guerrero (2016) which extended the work of Conover et al. (1981), and the third one was already used by Ross et al. (2011), which was introduced by Mood (1954).

1.3 Research Questions

1) Are the proposed Lepage-type Control Charts based on the WRS, F-K and S-R statistics, with the alignment around the median, robust for monitoring both the location and scale parameters under a variety of distributions and sample sizes?

2) Do the Control Charts mentioned in question 1 have a better performance (measured in terms of ARL), under a variety of sample sizes and distributions, than the Control Chart using the WRS and A-B statistics proposed by Chowdhury et al. (2015) for joint monitoring of location and scale?

3) Does the Lepage-type Control Chart based on the WRS and Mood statistics have a better performance (measured in terms of ARL), under a variety of sample sizes and distributions, than the Control Chart using the WRS and A-B statistics proposed by Chowdhury et al. (2015) for joint monitoring of location and scale?

Note that since Mood statistics is already known to be Distribution free, its robustness is not in doubt, hence, it is not included in question 1.

1.4 Hypotheses

1) The proposed Control Charts using the WRS, F-K and S-R statistics are robust (considering robustness as showing a maximal deviation of 10% from the expected ARLo) under a variety of distributions and sample sizes.

2) The mentioned Control Charts in hypotheses one has better performance than the Control Chart proposed by Chowdhury et al. (2015) for monitoring both location and scale.

3) The Lepage-type Control Chart based on the WRS and Mood statistics has a better performance than the Control Chart proposed by Chowdhury et al. (2015) for monitoring both location and scale.

1.5 Objectives

The aim of the present research is to propose several Control Charts able to monitor both Phase II location and scale parameters when data do not follow a known distribution, and that the performance of the Control Charts is better than the one which they will be compared.

Furthermore, the primary objective can be divided into specific targets to address the hypotheses mentioned in Section 1.4:

1) Measure the robustness (regarding how much variation exists between the In-Control ARL obtained via simulation and the desired ARLo) and define being robust as having a difference smaller or equal than 10%, of the proposed Control Charts using the F-K and S-R statistics.

2) Measure the performance (in terms of ARL) of the new Control Charts based on the F-K and S-R statistics and compare it to the one proposed in the article by Chowdhury et al. (2015), by using Monte Carlo simulations in a variety of sample sizes and distributions.

3) Measure the performance (in terms of ARL) of the new Control Chart based on the Mood statistics and compare it to the one proposed in the article by Chowdhury et al. (2015), by using Monte Carlo simulations in a variety of sample sizes and distributions.

1.6 Scope and limitations

As this thesis is an expansion of a previous work by Chowdhury et al. (2015), the scope is to identify, select and use statistics for monitoring the scale parameter to create a Control Chart that could perform better than the original one.

The robust origin of the A-B statistic makes the Control Chart able to function even when the assumption of normality cannot be made. The proposed statistics must also be able to work in the same way. Also, the Control Chart is especially useful for sustained changes over time, since it uses the Cumulative Sum methodology.

The study covers the use of the F-K, S-R and Mood statistics recommended in Guerrero (2016) as a better option than the A-B statistic for monitoring changes in the scale parameter under a variety of scenarios. Finally, it is known that change-point based control charts for location and scale exist. However, their computational complexity put them into another category and their performance is not included in this analysis.

The covered scenarios in which the study is limited are:

- Using only two-sample tests.
- A significance level of 0.05.
- Sample size. Reference samples from an IC process can be 30, 50, 100 and 150 observations while Phase II samples can be 5 or 11 observations.
- Distribution. The performance of the Control Charts is tested for two symmetric distributions, Normal and Cauchy, and a skewed distribution, Lognormal.
- Change size. All possible combinations are considered for changes in location (0, 0.25, 0.5, 0.75, 1, 1.5, 2, 3), scale (0.5, 1, 1.25, 1.5, 1.75, 2, 3) or both of them at the same time. For the Lognormal distribution, downward shifts in the location parameter also considered (-0.25, -0.5, -0.75, -1, -1.5, -2, -3).
- Average Running Length. Calculation of Control Limits and performance measuring is done for the values of ARL 500, 370 and 250.
- CUSUM reference value “k”. It can be either 0, 3 or 6 as proposed by Chowdhury et al. (2015). Further explanation of this value can be found in Chapter 5.
- The number of replicates using the Monte Carlo simulations is fixed to 50,000 as being the same amount used by Chowdhury et al. (2015) in the reference article.

1.7 Contribution

1.7.1 Scientific contribution

Three new CUSUM Control Charts based on the Lepage-type CUSUM Control Chart using the WRS and A-B statistics, proposed by Chowdhury et al. (2015) are created. By using the F-K, S-R and Mood statistics instead of the A-B, three new Lepage-type tests are also designed to manage changes in both location and scale parameters. The Control Charts performance is measured and compared with the reference Control Chart, and a decision table suggests in which scenarios each version of the Control Chart will perform best.

1.7.2 Contribution for practitioners

With the new charts put on the table, monitoring of changes in mean and variance for processes where the distribution that data follows is unknown should gain a boost in performance. There will be several Control Charts to choose, so a decision of picking the best Chart can be made for each scenario to be faced.

The application of robust and/or distribution-free Control Charts covers a broad range of fields; manufacturing processes can be an example, as Chowdhury et al. (2015) showed in their article using piston rings production data extracted from Montgomery (2001).

Chapter 2. Literature review

2.1. Research classification

The present work considers many sources, as it combines several different techniques to create a Control Chart that can be described by three characteristics; it is non-parametric, it's a Cumulative sum (CUSUM) type of chart, and it can monitor both the location and scale parameters.

Therefore, Tables 2.1 to 2.3 are a chronological compilation of all papers read to have an overview of the three topics covered. The rest of citations included in this study will be commented in this Chapter and can be found in the References section just before the Annexes.

Table 2.1. Non-parametric statistics related studies

Date of publication	Author/s	Described method
1952	Kruskal, W. H., & Wallis, W. A.	Population means difference analysis using ranking
1954	Mood, A. M.	Two-sample tests efficiency study
1957	Freund, J. E., & Ansari, A. R.	Two-way rank sum tests for variance
1960	Ansari, A. R., & Bradley, R. A.	Rank sum tests for dispersion
1976	Fligner, M. A., & Killeen, T. J.	Two-sample tests for scale parameter
1981	Conover, W. J., Johnson, M. E., & Johnson, M. M.	Homogeneity of variances study
1987	Park, C., Park, C., Reynolds, M. R., & Reynolds, M. R	Nonparametric monitoring of location parameter
2011	Ross, G. J., Tasoulis, D. K., & Adams, N. M.	Non-parametric data streams monitoring of location and scale
2014	Chowdhury, S., Mukherjee, A., & Chakraborti, S.	Distribution-free Cusum chart for monitoring location and scale
2016	Guerrero Serrano, A. J.	Homogeneity of variances study update

Table 2.2 Cumulative sum charts related studies

Date of publication	Author/s	Described method
1954	Page, E. S.	Cusum proposition
1983	Woodall, W. H.	One-sided Cusum procedures
1985	Lucas, J. M.	Cusum control schemes
1987	Hawkins, D. M.	Self-start cusum for location and scale
1990	Mcdonald, D.	Cusum based on sequential ranks
1995	Chang TC, Gan FF.	Variance monitoring cusum
1998	Hawkins, D. M., & Olwell, D. H.	Several Cusum charts
2008	Gan, F. F.	A Cusum chart
2011	Goel AL.	Several Cusum charts
2013	Mukherjee, A., Graham, M. A., & Chakraborti, S.	Distribution-free Cusum charts for location monitoring
2014	Chowdhury, S., Mukherjee, A., & Chakraborti, S.	Distribution-free Cusum chart for monitoring location and scale

Table 2.3 Location and scale monitoring related studies

Date of publication	Author/s	Described method
1971	Lepage, Y.	Wilcoxon and Ansari-Bradley statistics combination
2011	Ross, G. J., Tasoulis, D. K., & Adams, N. M.	Non-parametric data streams monitoring of location and scale
2013	McCracken AK., Chakraborti S., Mukherjee A	Normal process location and scale monitoring
2013	Mccracken, A., & Chakraborti	Mean and variance monitoring overview
2014	Chowdhury, S., Mukherjee, A., & Chakraborti, S.	Distribution-free Cusum chart for monitoring location and scale

2.2 Literature review

The main engine used for the search of the literature was the ProQuest Engineering Collection. This quest was mainly focused on the three topics mentioned before, using as key expressions: “nonparametric”, “distribution-free”, “CUSUM”, “cumulative sum”, “location and scale”, “mean and variance”, “joint monitoring”.

Since many papers have been published on the covered topics, the oldest articles are more of a primary reference to the presented question than a modern application study. The most useful and updated materials for this work range from 2011 to 2016.

2.2.1 Non-parametric approach

When a process distribution is far from the normal distribution, researchers defended using non-parametric, or better said distribution-free techniques. The first applications were focused on monitoring only one parameter, as shown by Park et al. (1987), focused only in the mean of continuous processes when the control value for the parameter is not specified. Nowadays, most non-parametric propositions are able to monitor both location and scale, and tests to detect both parameters are based on ranking techniques. Kruskal and Wallis (1952), firstly used ranks for detecting significant differences between the mean of two populations. The first assumption made was that the variation among the means reflected only random sampling from a population where individuals vary. The second assumption was that the variance of means of random samples of size $\boldsymbol{n_i}$ is

$$\frac{\boldsymbol{\sigma^2}}{\boldsymbol{n_i}},$$

where $\boldsymbol{\sigma^2}$ is the population variance. The estimate of $\boldsymbol{\sigma^2}$, which is based on the variation among sample means was compared with another estimator based on the variation within samples.

A couple of years later, Mood (1954) used the ranks for two-sample tests for the scale parameter. Basically, two samples $\boldsymbol{X}$ and $\boldsymbol{Y}$ of $\boldsymbol{m}$ and $\boldsymbol{n}$ independent observations from populations with continuous cumulative distribution functions $\boldsymbol{F}(\boldsymbol{u})$ and $\boldsymbol{G}(\boldsymbol{u})$ are considered. It is then required that the difference in medians of the two populations is known, and that the two samples may be adjusted to have the same location. Taking the location parameters to be zero, it is tested the hypotheses

$$\boldsymbol{G}(\boldsymbol{u}) \equiv \boldsymbol{F}(\boldsymbol{\theta u}),$$

with $\boldsymbol{\theta}$ different to 1. We order the two samples in a single array, assigning ranks from each end of the array towards the middle. From here, the test statistic is calculated and compared. Freund and Ansari (1957), Ansari and Bradley (1960) and Fligner and Killeen (1976) extended the work of scale parameter two-sample test ranking.

Modern trends have tended to combine the mentioned tests for non-parametric monitoring of mean or variance into single charts for joint monitoring. They are easier to use and have some advantages, as mentioned by McCracken et al. (2013). Cheng & Thaga (2006) classified two approaches that can be used; plotting two quality characteristics in the same chart, or using one plotting variable able to represent both location and spread. In the trend of using a single plotting variable, some authors have proposed to use a type of statistic firstly introduced by Lepage (1971), which combined the Wilcoxon and Ansari-Bradley tests for join monitoring of location and scale. In a similar way as explained by Kruskal and Wallis (1952) or Mood (1954), two samples from populations with continuous distribution functions $\boldsymbol{F}(\boldsymbol{x})$ and $\boldsymbol{G}(\boldsymbol{y})$ are considered. The hypotheses to be tested this time is

$$\boldsymbol{G}(\boldsymbol{x}) \equiv \boldsymbol{F}(\boldsymbol{x}) \,,$$

though this time the alternative is

$$\boldsymbol{G}(\boldsymbol{x}) \equiv \boldsymbol{F}(\boldsymbol{a} \cdot \boldsymbol{x} + \boldsymbol{b}) \,,$$

With $\boldsymbol{a}$ different to 1 and $\boldsymbol{b}$ different to 0. Ross et al. (2011), combining the Mood test for the scale and the Kruskal and Wallis test for the location, or again McCracken et al. (2013) are examples of the usage of this Lepage-type statistic.

For an overview of control charts able to monitor both location and scale, McCracken and Chakraborti wrote an article in 2013. There, charts are classified depending whether the standards are known or not, whether we use one-chart or two-chart monitoring schemes, and whether control charts are parametric or nonparametric.

2.2.2 CUSUM charts

Page (1954) was the father of the Cumulative sum (CUSUM) charts family. He classified two types of problems in processes, using variables $\boldsymbol{\theta}$ for the mean of the process and $\boldsymbol{k}$ for the number of observations; if $\boldsymbol{k}$ is small, large changes in $\boldsymbol{\theta}$ are rapidly detected, but small changes are detected slower. This case of large changes in the mean is easily detected by Shewhart charts. Then, a large value of $\boldsymbol{k}$ is necessary to detect rapidly small changes in $\boldsymbol{\theta}$. But, large changes in $\boldsymbol{\theta}$ will be noticed later, owing to the moving average damping the effect of one extreme observation.
CUSUM charts have shown usefulness in processes with small and sustained changes in parameters, like the latter case explained. They tend to have smaller ARLs than Shewhart charts in these scenarios, so CUSUM and Shewhart charts tend to complement each other.

Many different versions of CUSUM have been proposed, like the ones from Woodall (1983), where he presented a method for approximating the moments and percentage points of the Run Length distribution of one-sided CUSUM procedures for continuous random variables.

Then Hawkins (1987), proposed a self-start CUSUM technique for cases when the exact process mean and standard deviation are not known to be under control a priori, since CUSUM charts tended to assume known process parameters. Hawkins proposed using the running mean and standard deviation of all observations made on the process since start-up as substitution for the unknown true values of parameters mentioned before.

McDonald (1990) was able relate the CUSUM chart technique to non-parametric quality control approaches by using the sequential ranks of observations to detect abrupt changes in the sampling distribution to a stochastically larger distribution.

In case of wanting to have an overall vision on the existing CUSUM charts, one can read the work of Hawkins & Olwell (1998) for an application-oriented point of view, the work by Gan (2008) explaining a wide arrange of topics including strengths and weaknesses of CUSUM charts in front of Shewhart techniques with data sets demonstrations, process mean estimations, weighting of samples for signaling, or multivariate CUSUM schemes, among others.

2.3 Improvement opportunities

Chowdhury et al. (2015) combined the previous three topics to create a distribution-free CUSUM chart able to monitor both location and scale parameters. The statistics used are the Ansari & Bradley (1960) test for monitoring variance, and the Wilcoxon (1945) rank sum test for the process mean.

However, there is room for improvement, since there are several tests thought to have a better performance than the Ansari-Bradley for the scale. Guerrero (2016) extended the work of Conover et al. (1981) regarding the performance of many tests for equality of variances. The new tests proposed showed robustness for skewed distributions like the Lognormal, which is contemplated in the present work. The tests with better performance were the F-K test, proposed by Fligner and Killeen (1976), the T-G test by Talwar and Gentle (1977) and the S-R (squared rank) test already proposed by Conover et al. (1981). Another test to be added to the mix has already been introduced before, the Mood test (1954).

Chapter 3. Methodology

The following chapter describes how the steps have been followed to obtain the Control Charts proposed in this study. The first part covers the assumptions made regarding our model. Then, the three introduced tests are described as well as how they combine with the location parameter test to create a Lepage-type statistic**.**

3.1 Assumptions

- In this study, we consider all variables as independent and identically distributed random variables under control (IID). All elements in the sequence are independent of the random variables that came before.
- All variables are considered continuous, which has a direct impact on the experimentation since ranking of observations cannot involve ties.
- Sampling is considered in batch data.

3.2 Scale tests

Two of the tests proposed by Guerrero (2016) as the ones with the best performance for monitoring of scale are the F-K (Fligner and Killeen 1976) and S-R (Squared Ranks, Conover et al. 1981). Both of them adjust observations by subtracting the overall median, then ranking the absolute value to obtain the score. The farthest observations from the overall median obtain the biggest scores, as they come from the largest variance under the alternative hypothesis. Table 3.1 shows how to obtain the score for each test.

Table 3.1. Test scores

Test	Score function a_{nr}	R_{ij} as rank of
F-K	$\Phi^{-1}(\frac{1}{2} + \frac{r}{2*(N+1)})$	$\lvert X_{ij} - \text{median}(X_i) \rvert$
S-R	r^2	$\lvert X_{ij} - \text{median}(X_i) \rvert$

Where Φ^{-1} is the quantile function, the inverse of the cumulative distribution function for the normal distribution. The rank of observations is “r” and “N” is the sample size. Once obtained the score, the standardized chi-square with 1 degree of freedom approximation of the F-K and S-R statistic, named F-K^2 and S-R^2 for simplicity, are

$$\chi^2 = \sum_{i=1}^{2} n_i * (A_i - a)^2 / V^2,$$

where:

- “A_i” is the average score of the ith sample
- “a” is the overall mean of all N scores
- “V^2” is the sample variance of all N scores

The third test is the two-sample Mood test (Mood, A. M. 1954). As the previous tests are robust (since the distribution of their tests statistics depends on the functional form of the population distribution), the Mood test is truly distribution-free. The Mood test statistic is

$$\boldsymbol{M'} = \sum_{x_i \in S} (\boldsymbol{r}(\boldsymbol{x_i}) - \left(\frac{n+1}{2}\right))^2,$$

where “r” is the rank of the pooled sample “x_i” and “n” is the sample size. The mean and variance of the Mood statistic are

$$\boldsymbol{\mu_{M'}} = \boldsymbol{n_s} \frac{(\boldsymbol{n^2} - \boldsymbol{1})}{\boldsymbol{12}},$$

and

$$\boldsymbol{\sigma^2_{M'}} = \frac{\boldsymbol{n_s} * \boldsymbol{n_t} * (\boldsymbol{n} + \boldsymbol{1}) * (\boldsymbol{n^2} - \boldsymbol{4})}{\boldsymbol{180}},$$

where “n_s” is the size of the first sample, “n_t” is the size of the second sample, and “n” the global sample size.

Then, the Mood test is standardized and taken absolute value so it can detect both increased and decreases in the scale parameter, as shown in

$$\boldsymbol{M} = (\mathbf{M}' - \boldsymbol{\mu_{M'}}) / \sqrt{\boldsymbol{\sigma^2_{M'}}}. \tag{1}$$

3.3 Location test

The location test used is the same as the one proposed by Chowdhury et al. (2015), the WRS (Wilcoxon Rank-sum statistic). The WRS test score is

$$\boldsymbol{r} = \sum_{1}^{n_s} \boldsymbol{r_i},$$

the WRS test mean

$$\boldsymbol{\mu_{WRS}} = \boldsymbol{n_s} \frac{(\boldsymbol{n} + \boldsymbol{1})}{\boldsymbol{2}},$$

and WRS test variance

$$\boldsymbol{\sigma^2_{WRS}} = \frac{\boldsymbol{n_s} * \boldsymbol{n_t} * (\boldsymbol{n} + \boldsymbol{1})}{\boldsymbol{12}}.$$

In the same way as the Mood test, the WRS test is also standardized (as shown in equation 1). The variables’ names meaning is the same as used for the Mood statistic.

3.4 Construction of the Control Chart statistic

The paper by Chowdhury et al. (2015) proposes the use of the WRS test to detect changes in the location, plus the A-B test for changes in scale. Both statistics are combined in the same way as done by Ross et al. (2011). In this paper, a Lepage statistic was created by calculating the sum of both location and scale squared statistics as

$$\boldsymbol{LP = WRS^2 + AB^2},$$

where the chi-squared distribution with 2 degrees of freedom is used as the approximate distribution by the LP statistic. Ross used the Mood statistic for detecting changes in scale, then Chowdhury did so with the Ansari-Bradley statistic to create a cumulative sum-Lepage Chart (CL chart).

In this study, the same methodology is followed, substituting the Ansari-Bradley test for the ones proposed. As mentioned by Ross in his article, “the Lepage method seems the most common in the nonparametric hypothesis testing literature”. This way we obtain a Lepage-type statistic as a combination of the WRS test for monitoring the location and the F-K, S-R and Mood tests for detecting changes in the scale parameter.

For this particular case, the Fligner and Killeen and the Squared Rank statistics are already squared, since they are the Chi-squared version of the statistic. Therefore,

$$\boldsymbol{LP_{FK} = WRS^2 + FK},$$

$$\boldsymbol{LP_{SR} = WRS^2 + SR}$$

and

$$\boldsymbol{LP_{Mood} = WRS^2 + M^2}$$

represent the Lepage-type statistic using the F-K, S-R and Mood tests, respectively

3.5 Construction of CUSUM chart

Following the recommendation proposed by Chowdhury et al. (2015), the CUSUM Control Chart using a Lepage-type statistic is created following these steps:

- First, a reference sample $X_m = (X_1, X_2 \ldots X_m)$ of size “m” from an In-Control process is created. The reference sample can follow a standardized Normal, Cauchy or Lognormal distribution.
- Then, we originate $Y_{j,n} = (Y_{j1}, Y_{j2} \ldots Y_{jn})$, the “j-th” phase II sample of size “n”. These values must be created following the same distribution as the reference sample X_m.

- Following the 3.4 section, calculation of the Lepage statistic LP is performed.
- Once obtained LP, it is introduced for the plotting statistic

$$\mathbf{C}_{\boldsymbol{j}} = \mathbf{max}\,[\mathbf{0}, \mathbf{C}_{\boldsymbol{j}-\mathbf{1}} + (\mathbf{LP}^{\mathbf{2}} - \mathbf{2}) - \mathbf{k}], \qquad \mathbf{C}_{\mathbf{0}} = \mathbf{0}$$

- The value of $\mathbf{C}_{\boldsymbol{j}-\mathbf{1}}$ for the first iteration is $\mathbf{C}_{\mathbf{0}} = \mathbf{0}$. For the second iteration j=2, the value of $\mathbf{C}_{\boldsymbol{j}-\mathbf{1}}$ is the one obtained in the first iteration j=1, and so on. The value of “k” is a reference value that will be discussed further in Chapter 5.
- The plotting statistic $\mathbf{C}_{\boldsymbol{j}}$ is compared with upper control limit H_t as explained in the “GetARL” algorithm found in Appendix C.

3.6 Numerical example

The piston ring data used by Montgomery (2001) is proposed to track how our Charts work. For this example, the LP-M CUSUM Chart will be compared with the CL Chart suggested by Chowdhury et al. (2015). The data comprises 25 samples of 5 observations of an In-Control process, corresponding to the inside diameter of manufactured piston rings. These 125 observations shown in Table 3.2 are the reference data.

Table 3.2 Piston rings reference data

Sample number	Observations					Average	SD
1	74.030	74.002	74.019	73.992	74.008	74.010	0.0148
2	73.995	73.992	74.001	74.011	74.004	74.001	0.0075
3	73.988	74.024	74.021	74.005	74.002	74.008	0.0147
4	74.002	73.996	73.993	74.015	74.009	74.003	0.0091
5	73.992	74.007	74.015	73.989	74.014	74.003	0.0122
6	74.009	73.994	73.997	73.985	73.993	73.996	0.0087
7	73.995	74.006	73.994	74.000	74.005	74.000	0.0055
8	73.985	74.003	73.993	74.015	73.988	73.997	0.0123
9	74.008	73.995	74.009	74.005	74.004	74.004	0.0055
10	73.998	74.000	73.990	74.007	73.995	73.998	0.0063
11	73.994	73.998	73.994	73.995	73.990	73.994	0.0029
12	74.004	74.000	74.007	74.000	73.996	74.001	0.0042
13	73.983	74.002	73.998	73.997	74.012	73.998	0.0105
14	74.006	73.967	73.994	74.000	73.984	73.990	0.0153
15	74.012	74.014	73.998	73.999	74.007	74.006	0.0073
16	74.000	73.984	74.005	73.998	73.996	73.997	0.0078
17	73.994	74.012	73.986	74.005	74.007	74.001	0.0106
18	74.006	74.010	74.018	74.003	74.000	74.007	0.0070
19	73.984	74.002	74.003	74.005	73.997	73.998	0.0085
20	74.000	74.010	74.013	74.020	74.003	74.009	0.0080
21	73.982	74.001	74.015	74.005	73.996	74.000	0.0122
22	74.004	73.999	73.990	74.006	74.009	74.002	0.0074
23	74.010	73.989	73.990	74.009	74.014	74.002	0.0119
24	74.015	74.008	73.993	74.000	74.010	74.005	0.0087
25	73.982	73.984	73.995	74.017	74.013	73.998	0.0162

Then, Table 3.3 shows 15 samples of 5 observations corresponding to Phase II data. Therefore we can consider that n=5. Since the determination of control limits did not take into account this scenarios, simulations must be performed to obtain H_t for the cases of m=125, n=5 for k=0, 3 and 6, which are

$$\boldsymbol{H_t} = \{\mathbf{25.3515625}\ (\boldsymbol{k} = \mathbf{0}), \mathbf{7.51953125}\ (\boldsymbol{k} = \mathbf{3}), \mathbf{4.26953125}\ (\boldsymbol{k} = \mathbf{6})\}$$

Table 3.3 Piston rings Phase II data

Sample number	Observations					Average	SD
1	74.012	74.015	74.030	73.986	74.000	74.009	0.0440
2	73.995	74.010	73.990	74.015	74.001	74.002	0.0250
3	73.987	73.999	73.985	74.000	73.990	73.992	0.0150
4	74.008	74.010	74.003	73.991	74.006	74.004	0.0190
5	74.003	74.000	74.001	73.986	73.997	73.997	0.0170
6	73.994	74.003	74.015	74.020	74.004	74.007	0.0260
7	74.008	74.002	74.018	73.995	74.005	74.006	0.0230
8	74.001	74.004	73.990	73.996	73.998	73.998	0.0140
9	74.015	74.000	74.016	74.025	74.000	74.011	0.0250
10	74.030	74.005	74.000	74.016	74.012	74.013	0.0300
11	74.001	73.990	73.995	74.010	74.024	74.004	0.0340
12	74.015	74.020	74.024	74.005	74.019	74.017	0.0190
13	74.035	74.010	74.012	74.015	74.026	74.020	0.0250
14	74.017	74.013	74.036	74.025	74.026	74.023	0.0230
15	74.010	74.005	74.029	74.000	74.020	74.013	0.0290

The manufacturing process is represented in Figures 3.1 to 3.3 for the three choices of k, both for the LP-M Chart and the CL Chart. Data corresponding to the CL Chart is extracted from the article of Chowdhury.

The Control Charts for k=3 and k=6 show how the process stays In-control for the 11 first Phase II samples, then gets out of control in the 12th sample for the first time. The state remains OOC until the last sample, for both LP-M Chart and CL Chart by Chowdhury et al. (2015), and it had happened the same for Mukherjee and Chakraborti (2013). Note that the LP-M Chart data is shown in a combination of lines and little bubbles, while the CL Chart data is represented by straight lines.

Once seen the instant where the process gets out of control, it may be interesting to see whether this change was produced by a shift in the location, scale or both of them.

We perform a two-sided, two-sample WRS test for the 125 observations of reference data and the 12th sample of Phase II data to check if the change happens in the location parameter. This test yields a p-value p_{WRS}=0.0027 (the same value as Chowdhury). After it, we follow the same procedure but this time for a two-sided, two-sample Mood test for the scale parameter, finding a p-value p_{Mood}=0.0129.

Given a confidence level of 95%, both location and scale changes seem to happen, the first with a value of 0.27% and the second with a 1.29%. It can be concluded that there is strong evidence of a shift in the location and some evidence of a shift for the scale parameter.

If we repeat the same methodology with the reference data and the 13th sample of Phase II data, the results show p-values to be p_{WRS}=0.0016 and p_{Mood}=0.0098. The conclusions are very similar to the previous test, indicating a strong evidence for a shift in location and some evidence for the scale. Note in Figure 3.1 that the 13th observation gets OOC for k=0 (as with k=3, and k=6) but unlike the other charts, the 12th observation was still In-control. This delay of one observation in the k=0 chart may make us recommend the k=3 and k=6 charts as better options for detecting changes in both location and scale.

Last but not least, it is interesting to see results coming from the 15th Phase II data test. P-values for this scenario are p_{WRS}=0.0389 and p_{Mood}=0.3649, hence, there is some evidence of a change in the location but not at all for the scale this time.

Results from the CL Chart are very similar to the ones shown here, extracting the same conclusions as with the LP-M Chart. Both charts perform equally well for this set of data.

Figure 3.1 Piston ring data LP-M Chart and CL Chart comparison with k=0

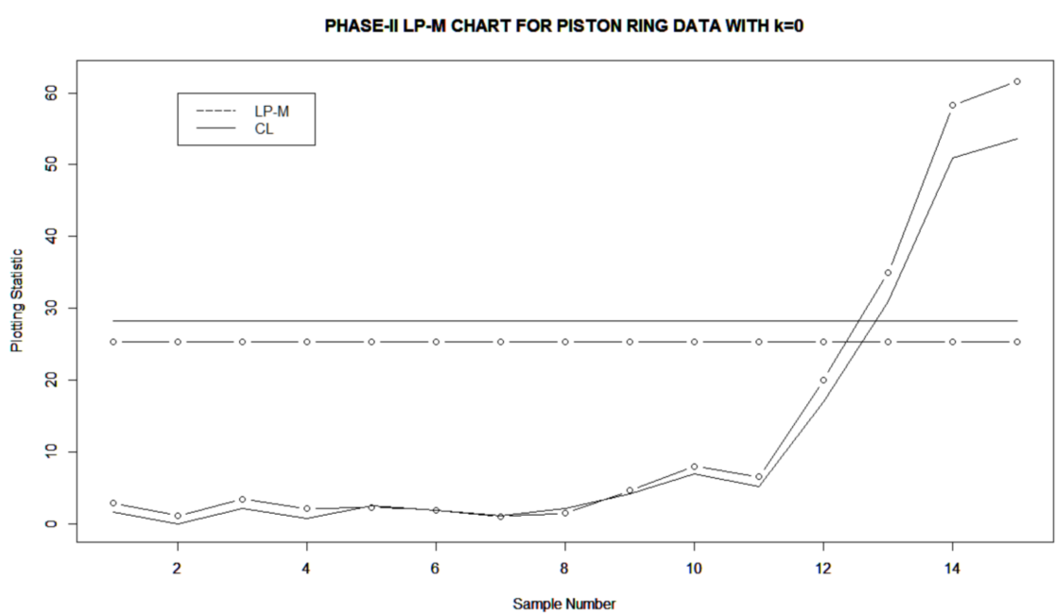


Figure 3.2 Piston ring data LP-M Chart and CL Chart comparison with k=3

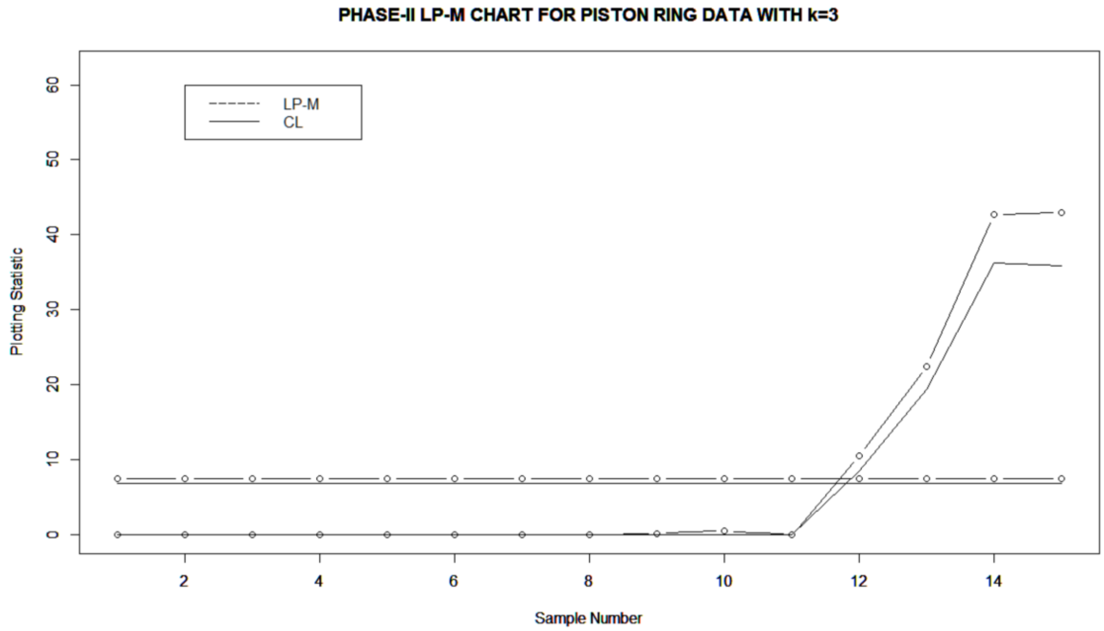
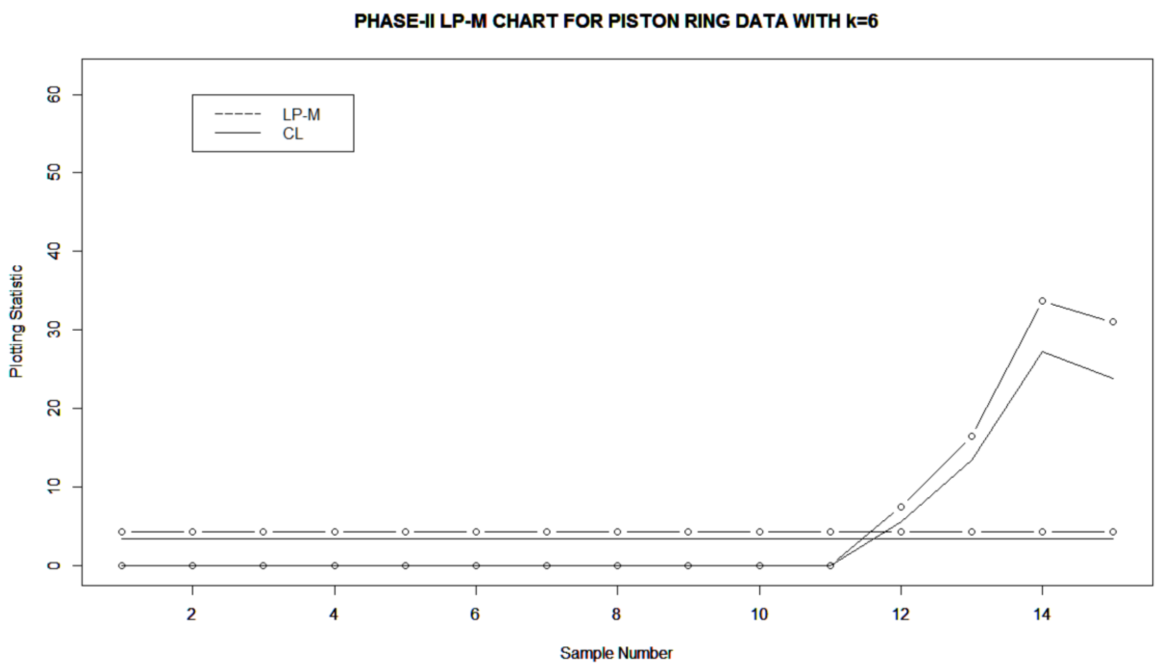


Figure 3.3 Piston ring data LP-M Chart and CL Chart comparison with k=6



Chapter 4. Simulation design and experimentation

The present chapter details all necessary steps to create the proposed Control Charts, plus the determination of the upper Control limits for practitioners. Last, the In-control performance of all charts is shown.

4.1 Computational calculation

All experimentation of the present study was based on Monte Carlo simulations running on RStudio, a powerful statistics software free to use.

Simulations were done to determine the Control Limits (H_t), the Average Run Length In-control (ARLo), the Run Length Standard Deviation (SDRL) and Run Length 5%, 25%, 50%, 75% and 95% Percentiles (RL P%) for every Control Chart proposed. Later, and to be discussed in Chapter 5, all mentioned variables are calculated again for Out-of-control states to measure the performance of each chart.

4.2 Used algorithms

Two main algorithms were created to conduct the study. First, the program called "GetARL" is an iterative calculation of Run Lengths for a fixed number of replicates:

1) For the desired Control Chart and each replicate, both location and scale statistics are calculated. Then, they are combined to create a Lepage-type statistic which is introduced in the CUSUM Chart methodology to obtain the CUSUM statistic (to be explained in section 4.3).
2) The CUSUM statistic is compared with an Upper Control Limit (UCL). If the value of the statistic is smaller than the Control Limit, a new iteration of the program is performed (during the same replicate), while the Run Length (RL) counter adds one unit. When the value of the CUSUM statistic is larger than the UCL, that particular replicate is over and the RL value is returned.
3) This process is repeated for each replicate; the value of ARL is obtained as the average of all RL, the SDRL is the standard deviation of all RL, and so on.

The second algorithm is called "GetH", an extension of the previous algorithm to obtain the Control Limits:

1) For the desired Control Chart and each replicate, we start with an "out of control" state. The UCL value is estimated between a minimum and a maximum. The first approximation of H_t is supposed to be the mean of both.

2) Then, while the state is OOC, the “GetARL” algorithm is called, calculating the value of the CUSUM statistic and compared with the first approximation of Ht for the desired number of replicates. This way we obtain the ARL. Three main possibilities exist then:

   a. First, the obtained value of ARL is smaller than the target value Ao (Ht is desired to be obtained for target values of Ao = 250, 370 and 500). If this happens, the value of Ht should be bigger than it was with the first iteration. “GetARL” is called again but this time, the minimum value of the UCL is the average between the minimum value of the UCL from the first iteration, and the value of H_t obtained from the first iteration.
   b. The second possibility is that the obtained value of ARL is bigger than the target one, Ao. The same methodology as the previous point is followed, but this time it is desired to reduce the value of H_t. The second iteration will be performed considering that the maximum value of the UCL is the average between the maximum value of the UCL from the first iteration, and the value of Ht obtained from the first iteration.
   c. The third possibility is that the calculation of the ARL gives us the desired target value Ao for an estimated UCL. The control limit Ht is then, equal to UCL.

3) In case a) or b) happens, more iterations of the algorithm are performed until c) happens.

The number of replicates is fixed to 50,000 since it is the value used by Chowdhury et al. (2015) to obtain their Control Limits. This way, direct comparison from this study and their paper can be made to analyze the obtained results. For more detail about the “GetH” and “GetARL” algorithms, consult the Annexes.

4.3 Determination of control limits

In the previous sections, all technical details have been specified to obtain H_t from the RStudio algorithm “GetH”. The explained methodology is followed then for the following scenarios:

- Reference sample m = 30, 50, 100 and 150
- Phase II sample n = 5 and 11
- In-control Average Run Length, ARLo = 500, 370 and 250
- CUSUM reference value k = 0, 3 and 6

All Control Limits are established using data coming from a standardized Normal distribution N(0,1). Combining all the previous scenarios, we obtain three tables from where we can consult triplets. A triplet is a combination of (m, n, k) for a given ARLo.

For example, consulting (50, 5, 0) for ARLo = 500 in the LP-FK CUSUM chart, means going to Table 4.1 and checking the case when k=0, m=50 and n=5, which gives us a Control Limit value of H_t=17.031.

Table 4.1 contains all Control Limits for the Control Chart using the Fligner and Killeen statistic for monitoring the scale parameter, called LP-FK CUSUM chart. Next is Table 4.2, which includes the same information when using the Squared Ranks test, in the Control Chart named as LP-SR CUSUM chart. Last, Table 4.3 includes Control Limits when using the Mood statistic for monitoring the scale parameter, naming the Control Chart the LP-M CUSUM chart.

Any needed value of H_t or ARL for a “k” between the values of 0 and 6 can be obtained by interpolating between the values of “k” used in the study. Values of “k” beyond 6 do not make much sense as Chowdhury et al. (2015) mentioned, since this value already cover shifts of 3 times the standard deviation of the CUSUM statistic $\mathbf{C}_{\boldsymbol{j}}$.

Checking the three tables, a pattern can be detected. Values of H_t are directly proportional to values of ARLo. Furthermore, when taking into account the value of “m” while fixing “n” and “k”, the value of Control Limits H_t tend to increase as “m” increases. But if we fix “m” and “k”, as we increase “n”, the Control Limits H_t tend to decrease. Finally, it’s been checked and affirmed that as “m” and “n” increase, Control Limits’ values increase until they stabilize, following the ratio

$$\boldsymbol{\lambda} = \frac{\boldsymbol{n}}{\boldsymbol{m}+\boldsymbol{n}}$$

as mentioned by Chowdhury et al. (2015).

Table 4.1. LP-FK CUSUM chart Control Limits

Chart parameter	Reference sample size	Test sample size	The charting constant (upper control limit): Ht		
k	m	n	Target ARLo = 250	Target ARLo = 370	Target ARLo = 500
0	30	5	11.875	12.875	13.721
		11	10.813	11.808	12.563
	50	5	14.500	16.046	17.031
		11	13.750	15.125	16.094
	100	5	18.250	20.563	22.375
		11	18.000	20.250	22.031
	150	5	20.500	23.250	25.438
		11	20.125	23.125	25.344
3	30	5	3.063	3.523	3.938
		11	3.500	3.969	4.328
	50	5	4.313	5.070	5.594
		11	4.000	4.563	4.922
	100	5	5.813	6.781	7.453
		11	4.875	5.563	6.094
	150	5	6.438	7.453	8.275
		11	5.375	6.188	6.797
6	30	5	0.004	0.409	0.723
		11	0.375	0.853	1.188
	50	5	1.203	1.929	2.477
		11	0.875	1.375	1.730
	100	5	2.625	3.550	4.227
		11	1.625	2.288	2.797
	150	5	3.188	4.190	4.953
		11	2.125	2.938	3.500

Table 4.2. LP-SR CUSUM chart Control Limits

Chart parameter	Reference sample size	Test sample size	The charting constant (upper control limit): Ht		
k	m	n	Target ARLo = 250	Target ARLo = 370	Target ARLo = 500
0	30	5	12.313	13.594	14.410
		11	11.063	12.125	12.938
	50	5	15.250	17.094	18.406
		11	14.250	15.750	16.969
	100	5	19.188	21.875	23.750
		11	18.750	21.250	23.250
	150	5	21.063	24.375	27.000
		11	21.125	24.500	27.102
3	30	5	2.949	3.457	3.883
		11	3.563	4.063	4.436
	50	5	4.289	5.004	5.594
		11	4.109	4.656	5.063
	100	5	5.688	6.563	7.242
		11	4.875	5.563	6.063
	150	5	6.156	7.094	7.836
		11	5.313	6.125	6.750
6	30	5	0.0005	0.337	0.694
		11	0.475	0.938	1.281
	50	5	1.156	1.898	2.453
		11	0.950	1.469	1.844
	100	5	2.500	3.313	3.991
		11	1.656	2.288	2.773
	150	5	2.914	3.875	4.531
		11	2.063	2.800	3.375

Table 4.3. LP-M CUSUM chart Control Limits

Chart parameter	Reference sample size	Test sample size	The charting constant (upper control limit): Ht		
k	m	n	Target ARLo = 250	Target ARLo = 370	Target ARLo = 500
0	30	5	12.313	13.625	14.336
		11	11.250	12.250	13.024
	50	5	15.250	17.031	18.213
		11	14.250	15.875	17.076
	100	5	19.000	21.750	23.664
		11	18.875	21.500	23.366
	150	5	21.000	24.344	26.830
		11	21.000	24.375	27.123
3	30	5	2.969	3.625	3.974
		11	3.297	3.719	4.033
	50	5	4.375	5.086	5.599
		11	3.922	4.422	4.832
	100	5	5.625	6.500	7.179
		11	4.781	5.469	5.993
	150	5	6.125	7.078	7.768
		11	5.313	6.063	6.668
6	30	5	0.002	0.506	0.887
		11	0.188	0.614	0.886
	50	5	1.275	1.919	2.480
		11	0.750	1.234	1.608
	100	5	2.453	3.336	3.932
		11	1.563	2.213	2.705
	150	5	2.906	3.813	4.500
		11	2.063	2.750	3.368

4.4 In-control performance under normality

Once obtained all control limits for each Control Chart (using the F-K, S-R, and Mood statistics for scale monitoring) and for each desired value of ARLo, the In-control performance was tested using Monte Carlo simulations using the “GetARL” algorithm. All results are summarized from Table 4.4 to Table 4.12, including the Average Run Length In-control (ARLo), the Run Length standard deviation (SDRL) and Run Length 5%, 25%, 50%, 75% and 95% percentiles.

It can be seen clearly that the In-control run length distribution is profoundly rightly skewed. For example, from Table 4.4 to Table 4.6, in all cases, the ARLo (around 500) is located between the third quartile and the 95th percentile, very far from the median, ranging from 76 to 273. The same happens for cases when ARLo=370 and 250, respectively.

The closest value between the Median and the ARLo happens when m=150, where the value of the 50th percentile is approximately half of the ARLo. The value of the 95th percentiles ranges from 3.3 to 4.2 times the ARLo for the three possible values of the In-control Average Run Length.

Though not linearly, as the reference sample “m” increases, the value of SDRL does decrease, and fixing “m” and “k”, SDRL decreases as the Phase II sample “n” increases from 5 to 11.

Looking at the percentiles, all of them (except the 95th) increase as the reference sample “m” goes up. There is no clear relationship if the Phase II sample “n” has an impact on the percentiles as we fix “m” and “k”. The same happens with “k”, there is no clear evidence that it has an impact on the percentiles when fixing “m” and “n”.

Finally, from the In-Control performance study, it can be concluded that all proposed Control Charts meet the In-control ARLo target value, at least for data under a Normal distribution. The In-control Average Run Length of the LP-FK and LP-SR CUSUM charts fluctuate with little deviation from its expected value (ARLo = 500, 370 or 250). The obtained confidence intervals are ARLo=[475.7, 507.7], ARLo=[365, 374.7] and ARLo=[245.04, 254.93] for the expected values of 500, 370 and 250. This accuracy is similar to the one obtained by Chowdhury et al. (2015), something to be expected since the same methodology was followed during the whole study.

Table 4.4. Normal distribution LP-FK CUSUM chart IC performance (ARLo=500)

m	n	H	ARLo	SDRLo	Fifth percentile	First quartile	Median	Third quartile	95th percentile
simulated values with K = 0									
30	5	13.721	500.397	3028.582	9	28	77	257	1720
30	11	12.563	504.164	2029.387	7	24	76	302	2060
50	5	17.031	501.816	4601.379	13	39	101	301	1679
50	11	16.094	495.340	2034.290	11	34	99	335	1965
100	5	22.375	496.159	1626.927	21	60	143	386	1876
100	11	22.031	497.982	1359.179	20	60	151	428	1959
150	5	25.438	504.933	2084.813	27	75	171	434	1811
150	11	25.344	496.551	1220.713	27	76	179	466	1905
simulated values with K = 3									
30	5	3.938	503.941	2533.813	5	32	105	347	1914
30	11	4.328	498.184	994.883	7	47	171	529	2050
50	5	5.594	501.594	2037.384	8	49	149	432	1889
50	11	4.922	498.505	1037.302	7	52	175	518	2005
100	5	7.453	498.120	955.855	13	77	218	542	1853
100	11	6.094	503.396	952.608	11	67	201	538	1965
150	5	8.275	503.504	790.634	16	94	251	593	1815
150	11	6.797	496.083	853.316	13	79	219	560	1861
simulated values with K = 6									
30	5	0.723	502.858	2852.749	5	32	103	327	1789
30	11	1.188	503.144	943.557	7	53	182	547	2049
50	5	2.477	497.584	1359.913	8	52	159	443	1930
50	11	1.730	496.368	980.468	8	56	180	527	2003
100	5	4.227	498.752	933.522	13	82	225	554	1836
100	11	2.797	496.690	921.123	11	68	202	530	1945
150	5	4.953	498.563	770.019	17	96	255	591	1779
150	11	3.500	500.075	876.814	14	82	225	564	1859

Table 4.5. Normal distribution LP-SR CUSUM chart IC performance (ARLo=500)

m	n	H	ARLo	SDRLo	Fifth percentile	First quartile	Median	Third quartile	95th percentile
simulated values with K = 0									
30	5	14.410	498.539	3491.671	10	31	85	272	1694
30	11	12.938	503.801	1959.036	7	25	79	316	2122
50	5	18.406	500.116	2012.977	15	46	117	341	1825
50	11	16.969	500.225	1711.067	12	38	111	369	2006
100	5	23.750	497.909	1429.376	25	70	165	429	1830
100	11	23.250	495.734	1262.468	23	66	165	458	1887
150	5	27.000	502.588	1293.732	33	89	196	475	1820
150	11	27.102	497.416	1003.877	32	86	203	518	1869
simulated values with K = 3									
30	5	3.883	502.518	2580.135	5	32	103	328	1833
30	11	4.436	499.965	952.225	7	51	180	539	2029
50	5	5.594	502.692	1625.736	9	54	159	448	1833
50	11	5.063	496.464	959.557	8	55	181	534	1994
100	5	7.242	501.961	918.853	15	86	234	567	1821
100	11	6.063	497.906	908.863	12	69	206	545	1925
150	5	7.836	503.179	755.957	17	101	269	606	1755
150	11	6.750	501.022	826.546	14	84	234	579	1851
simulated values with K = 6									
30	5	0.694	504.881	4760.369	5	31	102	317	1707
30	11	1.281	495.531	927.914	7	54	186	538	2008
50	5	2.453	496.783	1423.178	9	56	168	453	1855
50	11	1.844	501.714	956.936	9	58	189	547	1991
100	5	3.991	502.065	849.834	15	90	243	579	1816
100	11	2.773	504.983	892.711	11	73	209	552	1980
150	5	4.531	501.487	730.302	18	104	271	612	1738
150	11	3.375	495.209	839.255	14	85	233	566	1815

Table 4.6. Normal distribution LP-M CUSUM chart IC performance (ARLo=500)

m	n	H	ARLo	SDRLo	Fifth percentile	First quartile	Median	Third quartile	95th percentile
simulated values with K = 0									
30	5	14.336	478.537	2909.770	10	32	87	282	1751
30	11	13.024	495.568	1783.649	8	25	79	314	2080
50	5	18.213	500.329	2268.649	15	46	116	346	1829
50	11	17.076	507.547	1789.878	12	39	113	376	2057
100	5	23.664	500.985	1702.165	26	70	164	427	1860
100	11	23.366	491.942	1145.493	23	67	170	470	1921
150	5	26.830	496.497	1229.022	33	88	196	476	1815
150	11	27.123	507.663	993.782	32	88	208	528	1909
simulated values with K = 3									
30	5	3.974	498.227	2560.281	6	35	111	345	1817
30	11	4.033	504.127	1011.371	6	47	167	521	2107
50	5	5.599	498.694	1364.216	9	55	164	458	1900
50	11	4.832	504.299	998.190	8	54	179	530	2051
100	5	7.179	493.883	835.810	14	85	231	566	1801
100	11	5.993	493.924	874.438	12	72	208	544	1909
150	5	7.768	499.903	745.196	18	101	266	608	1745
150	11	6.668	493.673	810.724	14	84	233	573	1810
simulated values with K = 6									
30	5	0.887	475.724	1847.504	6	36	115	352	1812
30	11	0.886	498.528	964.733	7	50	175	527	2054
50	5	2.480	498.995	1378.776	10	60	175	475	1885
50	11	1.608	495.726	962.208	8	56	180	523	2016
100	5	3.932	493.777	799.055	15	90	243	581	1771
100	11	2.705	499.003	919.301	12	73	209	540	1935
150	5	4.500	498.909	693.196	19	107	273	616	1723
150	11	3.368	505.646	845.570	15	88	240	579	1853

Table 4.7. Normal distribution LP-FK CUSUM chart IC performance (ARLo=370)

m	n	H	ARLo	SDRLo	Fifth percentile	First quartile	Median	Third quartile	95th percentile
simulated values with K = 0									
30	5	12.875	374.129	2809.013	8	25	68	214	1318
30	11	11.808	368.715	1497.117	7	22	67	246	1506
50	5	16.046	365.065	1475.244	12	36	91	263	1361
50	11	15.125	366.280	1198.089	10	32	89	282	1471
100	5	20.563	370.617	1374.750	18	53	122	314	1330
100	11	20.250	370.083	858.949	18	51	126	345	1451
150	5	23.250	369.044	875.225	23	64	144	350	1357
150	11	23.125	371.037	812.011	23	65	151	376	1383
simulated values with K = 3									
30	5	3.523	372.033	1484.285	4	27	87	272	1428
30	11	3.969	372.001	707.505	6	40	134	405	1500
50	5	5.070	368.500	1088.545	7	39	119	329	1394
50	11	4.563	372.920	697.118	6	42	140	405	1507
100	5	6.781	374.166	694.790	11	62	170	414	1378
100	11	5.563	374.553	664.365	9	54	158	417	1455
150	5	7.453	366.476	560.276	12	71	188	438	1304
150	11	6.188	368.098	611.106	11	62	172	421	1366
simulated values with K = 6									
30	5	0.409	366.156	1392.956	4	27	86	266	1411
30	11	0.853	370.056	676.968	6	42	143	413	1473
50	5	1.929	367.158	979.902	7	42	126	342	1400
50	11	1.375	368.732	683.109	7	44	141	400	1463
100	5	3.550	372.271	737.155	11	64	172	419	1347
100	11	2.288	366.614	648.916	9	54	154	404	1423
150	5	4.190	365.220	550.811	13	74	193	442	1260
150	11	2.938	373.963	609.049	11	64	176	430	1394

Table 4.8. Normal distribution LP-SR CUSUM chart IC performance (ARLo=370)

m	n	H	ARLo	SDRLo	Fifth percentile	First quartile	Median	Third quartile	95th percentile
simulated values with K = 0									
30	5	13.594	374.180	2208.731	9	28	76	234	1310
30	11	12.125	368.537	1276.799	7	22	169	253	1531
50	5	17.094	374.695	1375.426	13	40	102	288	1366
50	11	15.750	366.263	1145.432	11	33	93	295	1441
100	5	21.875	371.598	890.952	22	61	140	349	1387
100	11	21.250	366.199	780.683	20	57	138	363	1401
150	5	24.375	366.088	768.710	28	74	162	377	1313
150	11	24.500	372.723	676.189	27	74	167	400	1349
simulated values with K = 3									
30	5	3.457	367.888	1401.752	4	27	86	266	1408
30	11	4.063	367.278	686.717	6	40	135	399	1474
50	5	5.004	365.202	955.475	7	42	125	340	1382
50	11	4.656	365.017	663.313	7	44	144	407	1435
100	5	6.563	370.946	641.193	11	66	179	426	1328
100	11	5.563	369.390	617.695	9	55	162	420	1456
150	5	7.094	369.779	532.434	13	77	200	452	1285
150	11	6.125	374.483	590.634	11	67	182	439	1365
simulated values with K = 6									
30	5	0.337	368.796	1697.212	4	27	82	255	1305
30	11	0.938	366.871	665.156	6	43	142	408	1469
50	5	1.898	366.746	923.194	7	44	131	351	1390
50	11	1.469	371.673	672.337	8	48	148	413	1456
100	5	3.313	366.669	584.199	12	69	182	430	1292
100	11	2.288	372.483	618.055	10	58	166	430	1399
150	5	3.875	373.354	521.643	14	80	205	461	1284
150	11	2.800	366.518	573.497	11	66	178	430	1341

Table 4.9. Normal distribution LP-M CUSUM chart IC performance (ARLo=370)

m	n	H	ARLo	SDRLo	Fifth percentile	First quartile	Median	Third quartile	95th percentile
simulated values with K = 0									
30	5	13.625	373.709	1694.463	9	29	79	246	1403
30	11	12.250	370.606	1166.228	7	23	70	260	1576
50	5	17.031	372.286	1259.143	14	41	103	291	1434
50	11	15.875	365.474	988.898	11	35	97	306	1492
100	5	21.750	369.202	894.232	22	61	139	348	1361
100	11	21.500	369.060	783.594	21	58	142	374	1395
150	5	24.344	369.629	735.095	28	74	163	380	1320
150	11	24.375	369.229	639.022	26	73	166	406	1351
simulated values with K = 3									
30	5	3.625	374.669	1390.608	5	29	93	284	1428
30	11	3.719	369.915	685.800	6	40	134	399	1517
50	5	5.086	368.752	846.883	8	45	132	360	1439
50	11	4.422	369.355	695.934	7	44	140	402	1479
100	5	6.500	370.322	593.764	12	68	181	430	1362
100	11	5.469	368.655	622.905	10	57	163	421	1389
150	5	7.078	371.916	515.902	14	79	203	458	1293
150	11	6.063	374.047	588.359	12	68	183	439	1365
simulated values with K = 6									
30	5	0.506	371.609	1415.961	5	30	93	279	1385
30	11	0.614	371.284	672.490	6	42	140	409	1508
50	5	1.919	366.796	824.934	8	47	136	364	1414
50	11	1.234	365.651	647.622	7	46	145	406	1460
100	5	3.336	374.461	622.943	12	71	188	442	1324
100	11	2.213	368.109	631.933	10	57	163	414	1401
150	5	3.813	373.106	505.734	14	82	209	463	1273
150	11	2.750	366.016	592.299	11	66	179	427	1321

Table 4.10. Normal distribution LP-FK CUSUM chart IC performance (ARLo=250)

m	n	H	ARLo	SDRLo	Fifth percentile	First quartile	Median	Third quartile	95th percentile
simulated values with K = 0									
30	5	11.875	254.514	1211.426	7	22	58	173	930
30	11	10.813	250.132	792.591	6	19	55	187	1051
50	5	14.500	247.516	810.436	10	30	73	200	914
50	11	13.750	252.676	727.108	9	27	73	217	998
100	5	18.250	248.457	593.925	15	43	98	239	903
100	11	18.000	250.578	525.050	14	42	100	255	940
150	5	20.500	250.309	488.442	19	52	113	259	894
150	11	20.125	246.026	403.329	18	51	115	272	889
simulated values with K = 3									
30	5	3.063	249.966	759.823	4	21	67	207	991
30	11	3.500	254.542	465.849	4	30	99	284	1007
50	5	4.313	248.261	602.269	5	30	88	236	949
50	11	4.000	246.302	438.797	5	32	101	279	958
100	5	5.813	246.252	416.833	8	44	118	282	888
100	11	4.875	253.572	419.053	7	40	114	240	959
150	5	6.438	248.456	367.289	9	51	133	302	857
150	11	5.375	250.068	375.977	8	46	124	299	921
simulated values with K = 6									
30	5	0.004	254.843	753.721	4	22	69	209	1021
30	11	0.375	250.249	431.661	5	31	101	285	988
50	5	1.203	250.884	738.660	6	31	90	240	945
50	11	0.875	250.714	434.943	5	34	105	285	976
100	5	2.625	246.637	420.717	8	45	121	284	884
100	11	1.625	246.188	398.710	7	40	113	284	926
150	5	3.188	245.053	354.904	9	51	133	300	848
150	11	2.125	251.091	397.425	8	46	123	295	905

Table 4.11. Normal distribution LP-SR CUSUM chart IC performance (ARLo=250)

m	n	H	ARLo	SDRLo	Fifth percentile	First quartile	Median	Third quartile	95th percentile
simulated values with K = 0									
30	5	12.313	247.036	1188.721	8	24	63	181	880
30	11	11.063	247.658	762.790	6	19	56	194	1027
50	5	15.250	254.386	2091.943	11	33	80	216	923
50	11	14.250	249.712	646.096	9	29	76	225	985
100	5	19.188	250.572	503.948	17	48	108	256	905
100	11	18.750	248.032	449.147	16	46	107	265	933
150	5	21.063	247.883	418.066	21	58	123	273	866
150	11	21.125	249.986	420.003	22	58	124	278	859
simulated values with K = 3									
30	5	2.949	250.676	883.320	4	21	65	199	960
30	11	3.563	249.692	441.976	5	31	99	282	976
50	5	4.289	250.910	678.691	6	32	92	243	936
50	11	4.109	250.455	427.449	6	34	106	285	972
100	5	5.688	254.100	420.399	9	48	126	297	902
100	11	4.875	250.532	390.342	7	42	117	297	936
150	5	6.156	249.647	336.627	10	54	140	312	862
150	11	5.313	248.239	364.950	9	47	126	301	885
simulated values with K = 6									
30	5	0.00045	264.486	950.706	4	23	70	210	1011
30	11	0.475	248.234	430.318	5	32	102	283	957
50	5	1.156	246.574	582.932	6	32	93	242	920
50	11	0.950	247.131	410.425	6	36	107	287	947
100	5	2.500	252.751	389.631	9	50	131	300	896
100	11	1.656	251.472	392.157	7	43	120	295	926
150	5	2.914	246.443	327.891	10	55	140	309	837
150	11	2.063	247.105	356.943	9	48	128	302	888

Table 4.12. Normal distribution LP-M CUSUM chart IC performance (ARLo=250)

m	n	H	ARLo	SDRLo	Fifth percentile	First quartile	Median	Third quartile	95th percentile
simulated values with K = 0									
30	5	12.313	246.074	926.318	8	25	64	187	936
30	11	11.250	254.930	735.925	6	20	59	200	1083
50	5	15.250	250.636	653.827	11	34	83	223	948
50	11	14.250	245.114	630.958	10	29	76	223	966
100	5	19.000	250.209	519.216	17	49	108	256	900
100	11	18.875	246.448	418.490	16	47	110	271	909
150	5	21.000	247.170	414.015	21	58	124	275	854
150	11	21.000	245.040	377.063	21	56	124	281	866
simulated values with K = 3									
30	5	2.969	251.755	1123.517	4	22	67	196	941
30	11	3.297	251.725	447.939	5	30	98	281	1002
50	5	4.375	254.197	697.149	6	33	95	251	966
50	11	3.922	249.856	418.694	6	35	104	284	977
100	5	5.625	249.790	403.735	9	47	125	294	887
100	11	4.781	247.044	384.204	7	42	117	289	920
150	5	6.125	251.383	345.441	10	55	141	311	862
150	11	5.313	252.725	377.565	8	48	129	306	899
simulated values with K = 6									
30	5	0.002	252.024	822.810	4	23	71	204	982
30	11	0.188	249.481	424.217	5	31	101	285	987
50	5	1.275	253.301	557.174	6	35	99	258	948
50	11	0.750	248.598	418.572	6	35	106	285	959
100	5	2.453	247.721	378.089	9	49	129	294	868
100	11	1.563	247.816	389.600	7	42	117	292	914
150	5	2.906	250.797	332.571	10	56	143	317	852
150	11	2.063	250.320	372.854	9	49	129	305	882

Chapter 5. Results and discussion

5.1 Out-of-control performance under non normality

The pattern followed here to measure the performance of the proposed Control Charts is the same one followed by Chowdhury et al. (2015) in his study. Three different distributions are considered: two of them are symmetric, the Normal and Cauchy distributions, and one is skewed, the Lognormal distribution.

Since this chapter is dedicated to measuring OOC performance, a change in the test sample must be introduced. As in the IC study data was generated as N(0,1), now Phase II observations $Y_{j,n}$ will follow an N(θ, δ) distribution. A total of 288 different scenarios for each distribution and each CUSUM chart are tried (all of them are shown for ARLo=500). In the same way as for the In-control study, the ARL, SDRL and Run Length percentiles are calculated. The parameters to be combined are:

- Location parameter θ = [0, 0.25, 0.5, 0.75, 1, 1.5, 2, 3]
- Scale parameter δ = [0.5, 1, 1.25, 1.5, 1.75, 2]
- Reference sample m = [50, 100]
- CUSUM reference value k = [0, 3, 6]

For the Lognormal distribution, the negative shifts are also considered since it is a skewed distribution (θ = [-3, -2, -1.5, -1, -0.75, -0.5, -0.25]). An upper limit has been set to 3000 for the Run Length counter following the methodology by Qiu (2013), to avoid simulations taking too long. This value has been fixed since no simulation performed by Chowdhury et al. (2015) gave an ARL larger than this. In case a simulation of this study results in a RL of 3000, the proposed Control Chart surely has a worse performance for that specific scenario than the one from the comparison study.

For the upcoming tables, a color code is introduced to make understanding of results easier:

- Shaded cells are those with a smaller ARL (they are faster) than the correspondent cells in the Chowdhury et al. study (2015)

- White cells with a triple asterisk (***) have a maximum of a 5% larger ARL than the correspondent cells in the Chowdhury et al. study (2015). It can imply that the proposed Chart had an equal or better performance than the existing chart, but computational calculations showed that result to be apparently slightly worse.

- White cells have a >5% larger ARL (they are slower) than the correspondent cells in the Chowdhury et al. study (2015).

- Also, each cell is read from left to right as; ARL, SDRL in brackets (), then the RL percentiles.

5.2 Performance of robust statistics LP-FK and LP-SR

The LP-FK and LP-SR CUSUM charts are the first two to be analyzed, with mixed results. In Chapter 4 both Control Charts were considered robust (considering robustness as showing a maximal deviation of 10% from the expected ARLo), at least when they are In-control. For the Out-of-control study though, they do not indicate robustness, at least for all scenarios and all distributions.

Tables 5.1 to 5.6 show the performance of the LP-FK and LP-SR CUSUM charts when there is no shift in the scale parameter, δ=1. Taking the row where θ=0 (no change in the location parameter), that is the scenario where no change is introduced in Phase II data. The results show robustness for both charts for the Normal distribution, as seen in Table 5.1 and Table 5.4. It can be confirmed by checking all values contained in the first row, when no change is introduced, which have less than a 10% deviation from the expected value of ARLo=500.

However, for Tables 5.2, 5.3, 5.5, and 5.6, the robustness is never completely accomplished. For example for Table 5.2 (LP-FK chart following a Cauchy distribution), the triplets (50, 5, 3) and (50, 5, 6) do not show robustness as it was defined in Chapter 1. For Table 5.3, no triplet (m, n, k) shows robustness for the Lognormal distribution of the LP-FK Chart, as all values have a more than 10% deviation from ARLo=500 when no change in the mean is introduced (θ=0).

Table 5.5 (LP-SR chart following a Cauchy distribution) only shows robustness for triplets (50, 5, 0) and (100, 5, 3), with all other combinations being non-robust. Last but not least, Table 5.6 let us see that no triplet is robust, ergo, the LP-SR CUSUM chart cannot be applied to data following a Lognormal distribution.

From this analysis, it can be concluded that the proposed LP-FK and LP-SR Control Charts do not show robustness in the same way as the chart proposed by Chowdhury et al. (2015) or the LP-M CUSUM chart (to be specifically analyzed later). Nevertheless, these two charts can be used in some specific scenarios.

Both LP-FK and LP-SR Charts could be used when data is known to follow a Normal distribution or the distribution is symmetric, but with some considerations in sample size. The LP-FK chart could be utilized when k=0 and/or m=100 under a Cauchy distribution. The LP-SR chart could also be used following a Cauchy distribution for specific scenarios (m=50, n=5, k=0) and (m=100, n=5, k=3). None of them is recommended for skewed data as shown in the Lognormal distribution (for a complete coverage of the LP-FK and LP-SR CUSUM charts Out-of-control performance, check the Appendix B).

Table 5.1. LP-FK chart Normal distribution OOC performance for δ=1

θ	m=50, n=5			m=100, n=5		
	MW-FK CUSUM chart			MW-FK CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
	δ = 1					
0	476.1 (2153.7) 13, 39, 101, 303, 1738	***496.6 (1466.2) 8, 49, 150, 437, 1926	***499.5 (1537.3) 9, 53, 158, 445, 1908	507.8 (2517.4) 21, 61, 144, 384, 1815	509.5 (1184.0) 13, 77, 216, 549, 1878	239.5 (364.4) 8, 43, 116, 279, 881
0.25	219.8 (1144.2) 8, 22, 47, 127, 727.056	286.4 (940.3) 4, 25, 79, 232, 1089	287.5 (883.8) 5, 28, 86, 249, 1114	163.8 (620.5) 12, 29, 57, 128, 551	***248.7 (527.8) 6, 36, 101, 260, 934	56.2 (86.8) 2, 11, 30, 68, 194
0.5	35.9 (152.4) 4, 9, 16, 31, 101	78.8 (268.0) 2, 8, 23, 65, 298	84.2 (321.6) 2, 9, 27, 72, 302	26.9 (37.8) 6, 12, 19, 31, 70	***58.6 (115.0) 2, 10, 25, 63, 214	16.4 (21.9) 1, 4, 10, 20, 54
0.75	10.3 (14.1) 2, 5, 8, 12, 25	18.8 (47.8) 1, 3, 8, 18, 66	22.1 (52.4) 1, 4, 9, 22, 79	10.4 (6.8) 3, 6, 9, 13, 23	14.7 (22.5) 1, 4, 8, 17, 49	6.2 (6.8) 1, 2, 4, 8, 19
1	5.3 (3.6) 2, 3, 5, 7, 12	6.2 (11.1) 1, 2, 3, 7, 19	7.6 (13.2) 1, 2, 4, 8, 25	5.7 (3.0) 2, 4, 5, 7, 11	5.2 (5.6) 1, 2, 4, 6, 15	3.1 (2.8) 1, 1, 2, 4, 8
1.5	2.5 (1.2) 1, 2, 2, 3, 5	1.8 (1.3) 1, 1, 1, 2, 4	2.0 (1.8) 1, 1, 1, 2, 5	2.6 (1.1) 1, 2, 2, 3, 5	1.8 (1.1) 1, 1, 1, 2, 4	1.4 (0.7) 1, 1, 1, 2, 3
2	1.5 (0.6) 1, 1, 1, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	1.6 (0.6) 1, 1, 2, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 2
3	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1

Table 5.2. LP-FK chart Cauchy distribution OOC performance for δ=1

θ	m=50, n=5			m=100, n=5		
	MW-FK CUSUM chart			MW-FK CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
	δ = 1					
0	457.3 (2779.8) 13, 39, 97, 282, 1611	397.0 (1042.7) 8, 48, 144, 386, 1509	401.6 (1113.3) 9, 51, 149, 396, 1519	480.5 (2221.1) 22, 61, 142, 375, 1744	464.6 (824.9) 13, 78, 213, 519, 1723	467.1 (793.2) 14, 81, 219, 534, 1697
0.25	373.1 (1930.4) 11, 32, 77, 232, 1314	368.0 (945.7) 7, 43, 126, 349, 1408	387.9 (1052.4) 8, 47, 137, 369, 1451	362.1 (1322.3) 18, 48, 107, 279, 1330	425.0 (813.5) 12, 68, 191, 474, 1561	424.7 (724.9) 12, 74, 201, 484, 1524
0.5	243.4 (1622.8) 8, 21, 45, 125, 805	297.5 (765.0) 5, 31, 95, 279, 1161	316.8 (1000.8) 6, 36, 105, 294, 1208	186.3 (751.3) 13, 30, 57, 131, 625	328.0 (638.1) 8, 49, 138, 350, 1246	345.1 (639.0) 9, 56, 154, 380, 1264
0.75	120.2 (801.3) 6, 13, 26, 60, 360	222.9 (682.2) 4, 19, 60, 183, 884	235.8 (683.6) 4, 23, 70, 202, 919	83.1 (988.7) 9, 18, 31, 59, 217	218.9 (464.5) 6, 30, 85, 224, 830	245.1 (464.9) 7, 37, 104, 265, 913
1	62.8 (665.0) 4, 9, 16, 31, 138	151.8 (551.5) 2, 11, 34, 110, 598	164.0 (581.2) 3, 14, 45, 133, 626	33.7 (158.0) 6, 12, 19, 32, 81	130.9 (300.0) 3, 16, 45, 125, 511	163.1 (345.0) 4, 23, 65, 168, 619
1.5	20.0 (1038.4) 3, 5, 8, 13, 32	60.2 (382.5) 1, 4, 10, 32, 210	76.0 (508.4) 1, 5, 16, 50, 273	11.4 (9.5) 4, 7, 9, 13, 25	36.9 (115.1) 2, 5, 12, 31, 135	62.4 (144.7) 2, 8, 23, 60, 238
2	49.2 (9507.5) 2, 3, 5, 7, 15	19.7 (136.9) 1, 2, 4, 10, 58	29.8 (134.6) 1, 3, 6, 19, 107	6.6 (3.5) 3, 4, 6, 8, 13	***10.4 (38.6) 1, 3, 5, 10, 32	***22.9 (58.7) 1, 3, 8, 21, 86
3	3.5 (33.2) 2, 2, 3, 4, 6	3.7 (29.3) 1, 1, 2, 3, 8	6.5 (85.5) 1, 1, 2, 4, 18	3.7 (1.4) 2, 3, 3, 4, 6	***2.6 (2.5) 1, 1, 2, 3, 6	***4.3 (11.7) 1, 1, 2, 4, 13

Table 5.3. LP-FK chart Lognormal distribution OOC performance for δ=1

θ	m=50, n=5			m=100, n=5		
	MW-FK CUSUM chart			MW-FK CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
δ = 1						
-3	***2.0 (0.2) 2, 2, 2, 2, 2	1.0 (0.2) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 2	***2.2 (0.4) 2, 2, 2, 2, 3	***1.1 (0.3) 1, 1, 1, 1, 2	***1.1 (0.3) 1, 1, 1, 1, 2
-2	***2.7 (0.7) 2, 2, 3, 3, 4	***1.7 (0.8) 1, 1, 2, 2, 3	2.0 (1.6) 1, 1, 2, 2, 5	***3.1 (0.7) 2, 3, 3, 3, 4	***1.9 (0.8) 1, 1, 2, 2, 3	2.3 (1.5) 1, 1, 2, 3, 5
-1.5	***3.8 (1.2) 2, 3, 4, 4, 6	3.2 (2.6) 1, 2, 3, 4, 8	6.8 (12.6) 1, 2, 4, 8, 21	4.5 (1.2) 3, 4, 4, 5, 7	3.5 (2.1) 1, 2, 3, 4, 7	7.8 (10.3) 1, 3, 5, 9, 24
-1	***7.3 (3.7) 3, 5, 7, 9, 14	21.2 (71.9) 2, 4, 9, 20, 71	74.1 (200.0) 2, 8, 23, 63, 279	8.6 (3.3) 4, 6, 8, 10, 15	20.9 (37.9) 2, 6, 11, 23, 68	133.7 (411.1) 3, 17, 47, 124, 481
-0.75	***13.8 (16.4) 5, 7, 11, 16, 31	125.4 (407.7) 3, 12, 33, 96, 483	283.9 (545.2) 5, 27, 84, 257, 1358	15.2 (8.3) 6, 10, 13, 18, 30	158.5 (461.4) 5, 20, 55, 144, 598	858.4 (2779.8) 12, 77, 238, 705, 3281
-0.5	***48.3 (176.8) 7, 14, 22, 41, 139	568.6 (1641.2) 9, 52, 169, 502, 2232	693.2 (878.5) 14, 95, 305, 895, 3001	42.2 (93.2) 11, 19, 29, 46, 107	1104.6 (2288.5) 22, 138, 416, 1137, 4294	2541.0 (5376.6) 50, 328, 977, 2651, 9726
-0.25	194.7 (573.1) 13, 31, 65, 162, 719	762.1 (1805.9) 16, 97, 287, 765, 2923	707.1 (854.4) 19, 117, 346, 922, 3001	219.8 (493.9) 22, 49, 96, 213, 781	1232.5 (2119.8) 34, 206, 573, 1401, 4497	1479.3 (2750.3) 37, 232, 651, 1616, 5513
0	239.3 (864.5) 11, 34, 81, 205, 857	309.0 (967.2) 6, 36, 107, 290, 1165	284.7 (490.2) 6, 37, 109, 295, 1197	289.3 (909.9) 19, 53, 119, 281, 1014	309.4 (549.5) 9, 51, 139, 344, 1150	321.5 (608.1) 10, 53, 146, 356, 1166
0.25	80.2 (273.0) 5, 13, 28, 65, 279	71.0 (154.6) 2, 10, 28, 71, 267	72.9 (153.6) 2, 11, 29, 74, 271	63.7 (132.7) 7, 18, 33, 65, 207	65.0 (116.7) 3, 13, 33, 75, 227	68.6 (109.8) 3, 14, 35, 81, 241
0.5	17.8 (39.5) 3, 6, 10, 19, 51	18.9 (39.4) 1, 4, 9, 21, 66	20.2 (36.9) 1, 4, 10, 22, 71	15.3 (14.2) 3, 7, 12, 19, 39	16.9 (23.4) 1, 4, 10, 21, 55	19.2 (26.0) 1, 5, 11, 24, 64
0.75	6.8 (6.2) 2, 3, 5, 8, 17	6.5 (9.5) 1, 2, 4, 8, 21	7.3 (10.6) 1, 2, 4, 9, 24	6.9 (4.5) 2, 4, 6, 9, 15	6.0 (6.5) 1, 2, 4, 8, 18	6.9 (7.9) 1, 2, 4, 9, 21
1	3.8 (2.5) 1, 2, 3, 5, 8	3.1 (3.3) 1, 1, 2, 4, 8	3.4 (3.7) 1, 1, 2, 4, 10	4.0 (2.2) 1, 2, 4, 5, 8	2.9 (2.5) 1, 1, 2, 4, 8	3.3 (3.1) 1, 1, 2, 4, 9
1.5	1.9 (0.9) 1, 1, 2, 2, 3	1.4 (0.7) 1, 1, 1, 2, 3	1.4 (0.9) 1, 1, 1, 2, 3	2.0 (0.9) 1, 1, 2, 2, 4	1.4 (0.7) 1, 1, 1, 2, 3	1.4 (0.8) 1, 1, 1, 2, 3
2	1.3 (0.5) 1, 1, 1, 2, 2	1.0 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.4 (0.5) 1, 1, 1, 2, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1

Table 5.4. LP-SR chart Normal distribution OOC performance for δ=1

θ	m=50, n=5			m=100, n=5		
	MW-SR CUSUM chart			MW-SR CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
δ = 1						
0	***527.7 (2682.0) 15, 45, 116, 346, 1850	487.6 (1462.6) 9, 53, 159, 442, 1871	498.5 (2372.8) 9, 56, 167, 460, 1865	481.3 (1399.8) 25, 69, 162, 420, 1799	508.6 (911.9) 15, 84, 233, 562, 1878	498.3 (853.7) 15, 88, 239, 569, 1801
0.25	231.6(1483.9) 9, 24, 53, 141, 792	***297.7 (1019.6) 5, 28, 84, 248, 1129	288.4 (839.6) 5, 30, 90, 255, 1104	163.3 (536.6) 14, 33, 63, 137, 555	***251.5 (495.3) 7, 39, 107, 269, 936	***257.4 (506.4) 7, 43, 116, 288, 937
0.5	38.0 (132.9) 5, 10, 18, 33, 109	80.2 (305.0) 2, 9, 24, 67, 304	83.0 (225.1) 2, 10, 28, 75, 314	28.6 (32.1) 7, 13, 21, 34, 74	59.6 (112.7) 2, 11, 27, 66, 215	65.9 (112.8) 3, 12, 32, 75, 235
0.75	11.2 (11.8) 3, 5, 9, 13, 27	19.5 (59.7) 1, 3, 8, 19, 68	22.6 (51.1) 1, 4, 10, 23, 80	11.3 (7.2) 4, 7, 10, 14, 24	***15.3 (24.0) 1, 4, 8, 18, 51	19.1 (30.4) 1, 4, 10, 23, 65
1	5.9 (3.8) 2, 3, 5, 7, 13	6.3 (10.8) 1, 2, 3, 7, 20	7.8 (12.7) 1, 2, 4, 9, 26	6.2 (3.2) 2, 4, 6, 8, 12	5.5 (6.2) 1, 2, 4, 7, 16	6.9 (8.3) 1, 2, 4, 9, 22
1.5	2.7 (1.2) 1, 2, 2, 3, 5	1.9 (1.3) 1, 1, 1, 2, 4	2.1 (1.9) 1, 1, 1, 2, 5	2.9 (1.2) 1, 2, 3, 4, 5	1.8 (1.1) 1, 1, 2, 2, 4	2.0 (1.5) 1, 1, 1, 2, 5
2	1.7 (0.7) 1, 1, 2, 2, 3	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	1.9 (0.7) 1, 1, 2, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
3	1.1 (0.3) 1, 1, 1, 1, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.1 (0.3) 1, 1, 1, 1, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1

Table 5.5. LP-SR chart Cauchy distribution OOC performance for δ=1

θ	m=50, n=5			m=100, n=5		
	MW-SR CUSUM chart			MW-SR CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
δ = 1						
0	480.3 (3799.7) 15, 45, 113, 331, 1734	357.5 (551.7) 9, 53, 153, 398.25, 1472	357.2 (544.9) 9, 55, 155, 402, 1455	420.7 (749.9) 25, 70, 161, 405, 1749	521.6 (889.7) 16, 93, 251, 604.25, 1851	442.9 (568.9) 16, 91, 235, 551, 1664
0.25	415.6 (2323.3) 13, 37, 90, 265, 1484	334.8 (535.7) 8, 47, 137, 366, 1398	339.0 (529.8) 8, 50, 142, 381, 1380	329.3 (649.3) 21, 54, 119, 301, 1312	465.2 (801.5) 14, 81, 221, 531, 1666	412.9 (557.8) 13, 79, 212, 499, 1567
0.5	256.7 (1630.7) 9, 23, 49.5, 138, 860	279.7 (493.4) 5, 32, 100, 287, 1224	283.7 (485.2) 6, 36, 107, 298, 1188	170.1 (411.0) 15, 32, 61, 139, 624	356.7 (660.8) 10, 55, 156, 394, 1338	316.9 (451.3) 9, 57, 156, 384, 1168
0.75	140.8 (1509.2) 7, 14, 27, 63, 372	203.0 (419.6) 3, 19, 61, 186, 885	214.2 (418.9) 4, 23, 71, 208, 904.0	71.6 (209.6) 10, 19, 33, 61, 206	229.9 (508.8) 6, 31, 88, 235, 889	223.7 (365.4) 6, 37, 100, 245, 843
1	71.5 (835.6) 5, 10, 17, 33, 144	135.0 (331.0) 2, 10, 33, 107, 584	149.9 (339.5) 3, 14, 43, 130, 627	31.8 (77.4) 7, 13, 20, 32, 82	131.1 (307.2) 4, 15, 44, 126, 522	147.6 (266.1) 4, 22, 61, 160, 565
1.5	16.1 (437.2) 3, 5, 8, 13, 33	51.2 (187.2) 1, 4, 10, 29, 197	65.0 (204.0) 1, 5, 14, 45, 258	11.5 (10.8) 4, 7, 9, 14, 25	34.0 (104.0) 2, 5, 11, 28, 124	51.3 (115.9) 2, 7, 18, 49, 203
2	7.6 (164.2) 2, 4, 5, 8, 15	18.0 (98.9) 1, 2, 4, 9, 53	27.2 (119.0) 1, 2, 5, 16, 95	6.5 (3.5) 3, 4, 6, 8, 13	9.2 (36.1) 1, 2, 4, 8, 27	16.9 (40.9) 1, 3, 6, 16, 62
3	3.3 (1.7) 2, 2, 3, 4, 6	3.4 (28.5) 1, 1, 2, 3, 7	4.9 (21.8) 1, 1, 2, 3, 13	3.5 (1.3) 2, 3, 3, 4, 6	2.3 (2.2) 1, 1, 2, 3, 5	3.0 (6.5) 1, 1, 2, 3, 8

Table 5.6. LP-SR chart Lognormal distribution OOC performance for δ=1

θ	m=50, n=5			m=100, n=5		
	MW-SR CUSUM chart			MW-SR CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
δ = 1						
-3	***2.0 (0.2) 2, 2, 2, 2, 2	1.1 (0.2) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 2	***2.2 (0.4) 2, 2, 2, 2, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
-2	***2.8 (0.7) 2, 2, 3, 3, 4	***1.7 (0.8) 1, 1, 2, 2, 3	2.0 (1.6) 1, 1, 1, 2, 5	3.2 (0.7) 2, 3, 3, 4, 4	1.8 (0.8) 1, 1, 2, 2, 3	2.1 (1.4) 1, 1, 2, 3, 5
-1.5	***4.0 (1.3) 2, 3, 4, 5, 6	3.1 (2.4) 1, 2, 3, 4, 7	6.5 (11.3) 1, 2, 4, 7, 21	4.6 (1.3) 3, 4, 4, 5, 7	3.3 (2.0) 1, 2, 3, 4, 7	6.9 (8.8) 1, 2, 4, 8, 21
-1	***7.6725 (3.77713238907265) 4, 5, 7, 9, 14	19.1 (46.7) 2, 4, 8, 18, 65	72.8 (375.8) 2, 8, 22, 58, 251	8.9 (3.4) 5, 7, 8, 11, 15	18.4 (31.3) 2, 5, 10, 21, 59	94.2 (249.7) 3, 13, 36, 91, 342
-0.75	***14.2 (18.3) 5, 8, 11, 16, 31	116.5 (526.6) 3, 11, 30, 85, 419	333.4 (1349.6) 4, 25, 78, 238, 1241	15.5 (7.9) 7, 10, 14, 19, 30	121.7 (298.6) 4, 17, 45, 116, 452	541.3 (1678.9) 9, 53, 161, 461, 2055
-0.5	***45.5 (105.8) 8, 14, 23, 42, 134	511.5 (1608.8) 8, 48, 153, 457, 2013	947.7 (2766.7) 13, 90, 284, 833, 3676	41.7 (439.5) 11, 19, 29, 45, 99	845.8 (1696.6) 17, 107, 328, 887.25, 3281	1788.4 (3907.5) 35, 229, 683, 1863, 6799
-0.25	187.3 (553.5) 14, 32, 67, 165, 672	698.5 (1450.9) 16, 99, 282, 736, 2634	889.5 (2026.9) 19, 116, 336, 884, 3395	***196.8 (374.1) 23, 50, 95, 202, 674	1095.7 (1778.2) 33, 196, 529, 1283, 3952	1274.3 (2277.7) 36, 217, 584, 1425, 4632
0	244.8 (623.7) 13, 39, 92, 232, 892	282.1 (617.5) 6, 36, 106, 282, 1094	309.7 (748.9) 6, 38, 111, 302, 1178	276.0 (500.8) 22, 61, 134, 304, 966	308.6 (538.8) 9, 53, 142, 348, 1135	302.9 (555.8) 9, 54, 144, 344, 1089
0.25	86.8 (289.7) 6, 15, 32, 75, 314	71.8 (186.6) 2, 11, 29, 72, 263	72.6 (163.6) 2, 11, 30, 75, 270	69.3 (127.5) 8, 20, 37, 72, 226	66.1 (101.2) 3, 13, 34, 78, 233	69.3 (106.3) 3, 14, 37, 83, 237
0.5	19.5 (34.7) 3, 7, 12, 21, 57	19.2 (33.8) 1, 4, 9, 21, 67	20.8 (36.3) 1, 4, 10, 23, 72	17.4 (16.4) 4, 8, 13, 21, 43	17.7 (24.2) 1, 4, 10, 22, 58	19.9 (26.0) 1, 5, 12, 25, 65
0.75	7.5 (7.0) 2, 4, 6, 9, 18	6.8 (9.5) 1, 2, 4, 8, 22	7.5 (10.3) 1, 2, 4, 9, 25	7.7 (4.8) 2, 4, 7, 10, 17	6.3 (6.8) 1, 2, 4, 8, 19	7.4 (8.3) 1, 2, 5, 9, 23
1	4.2 (2.6) 1, 2, 4, 5, 9	3.2 (3.2) 1, 1, 2, 4, 9	3.6 (3.9) 1, 1, 2, 4, 10	4.5 (2.3) 2, 3, 4, 6, 9	3.1 (2.6) 1, 1, 2, 4, 8	3.5 (3.3) 1, 1, 2, 4, 10
1.5	2.1 (1.0) 1, 1, 2, 3, 4	1.4 (0.8) 1, 1, 1, 2, 3	1.5 (0.9) 1, 1, 1, 2, 3	2.3 (0.9) 1, 2, 2, 3, 4	1.4 (0.7) 1, 1, 1, 2, 3	1.5 (0.9) 1, 1, 1, 2, 3
2	1.4 (0.6) 1, 1, 1, 2, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.6 (0.6) 1, 1, 2, 2, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
3	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1

5.3 Performance of the distribution-free LP-M Chart

The same three distributions and 288 different scenarios explained before are tried to measure the performance of the Control Chart using the Mood statistic to monitor changes in the scale parameter.

First of all, this section has been separated from the previous two Control Charts because the LP-M Chart is distribution-free, showing robustness in the Normal, Cauchy and Lognormal distributions. This fact can be checked by viewing the results of the ARL for the case of both scale and location invariance (θ=0 and δ=1). In Tables 5.7 to 5.9 the ARL shows to have less than a 10% deviation from the expected value of ARLo=500.

Once satisfied the hypotheses of being distribution-free, the performance of the LP-M Chart can be compared to the one presented by Chowdhury et al. (2015) in his paper.

5.3.1 Normal distribution

The results from Table 5.7 can conclude that the LP-M Chart has an overall better performance than the CL Chart. Out of 288 possible scenarios, the proposed chart outperforms the CL chart in 252 cases. Table 5.7 shows that the Mood statistic has a bad performance for downward shifts (δ=0.5). For this case, the CL chart is better in 27 cases, while the LP-M chart is better in 21. The previous would be the only case in which the usage of the Ansari-Bradley statistic is recommended to create a cumulative sum chart. In practice, most of the times variance increases over time. And for $\delta \geq 1$, the LP-M chart outperforms in 233 out of 240 different scenarios.

If we take a look at the triplet (50, 5, 6), we can see that for a 25% increase in the scale (δ=1 to δ=1.25) while θ=0, there is an 86% decrease in the ARL (from 495.7 to 69.7). In case there is a 25% increase in the location parameter (θ=1 to θ=1.25) while δ=1, the decrease in the ARL is around 41% (from 495.7 to 294.7). Combining both increases, the Average Run Length is reduced from 495.7 to 52.8, almost a 90%. Therefore, the LP-M is useful to detect changes in both location and scale parameters, having a stronger performance in terms of ARL when a change is introduced in both of them.

The grade of improvement keeps more or less the same for the rest of triplets. For large shifts in the location parameter, a 150%, the detection speed of the Control Chart is almost instantaneous (it takes 3 observations or less) regardless of the existence of a shift in the scale parameter.

The same analysis can be done for the SDRL and the RL percentiles. In case we take a look at the same examples used before for the standard deviation, the decrease is more or less similar. There is a 91% reduction for $\delta=1$ to $\delta=1.25$ with $\theta=0$, a 30% reduction $\theta=1$ to $\theta=1.25$ with $\delta=1$, and a combined reduction of 93% when both location and scale parameter increase. For the Run Length percentiles, the results are 88%, 41% and 91% comparing the 95th percentile of all cases. As it happened in the previous study, the OOC run length distributions are rightly skewed (can be checked in Tables 5.7 to 5.9).

5.3.2 Cauchy distribution

The performance of the LP-M chart is slightly less favorable for a heavy-tailed distribution like the Cauchy than the Normal distribution. In this case, the Mood statistic based chart outperforms the Ansari-Bradley statistic chart in 179 out of 288 scenarios, which is still more than 60% of the times.

For this case, while $\delta=0.5$ or $\delta=1$, CL chart is better in 62 cases while the LP-M chart in 34 cases. Therefore, the A-B statistic should be recommended in cases where there are downward or no shifts in the scale parameter. When there is an increase in the manufacturing process variance, the Mood statistic is faster (for $\delta \geq 1.25$, the LP-M chart wins in 75% of the cases).

Using the same triplet and same scenarios than the Normal distribution analysis, the results are:

- $\delta=1$ to $\delta=1.25$ while $\theta=0$ → 57% decrease in ARL, 62% decrease in SDRL and 57% decrease in 95th percentile

- $\theta=1$ to $\theta=1.25$ while $\delta=1$ → 4% decrease in ARL, 8% decrease in SDRL and 4.5% decrease in 95th percentile

- $\delta=1$ to $\delta=1.25$ and $\theta=1$ to $\theta=1.25$ → 59% decrease in ARL, 64% decrease in SDRL and 58% decrease in 95th percentile

The pattern is more or less similar to the Normal distribution analysis, but now detection of changes is slower under the heavy-tailed distribution.

5.3.3 Lognormal distribution

The last comparison is done for a skewed distribution, in this case the Lognormal(θ,δ). As before, the same analysis is done for the (50, 5, 6) triplet, shown in the three cases studied:

- δ=1 to δ=1.25 while θ=0 → 87% decrease in ARL, 94% decrease in SDRL and 88% decrease in 95^{th} percentile

- θ=1 to θ=1.25 while δ=1 → 43% decrease in ARL, 56% decrease in SDRL and 41% decrease in 95^{th} percentile

- δ=1 to δ=1.25 and θ=1 to θ=1.25 → 90% decrease in ARL, 95% decrease in SDRL and 90% decrease in 95^{th} percentile

The detection speed is the highest for the Lognormal distribution, having slightly lower values of ARL, SDRL and RL percentiles than the Normal distribution.

Since now the study of negative and positive shifts in the location parameter (θ) is done separately, there are a total of 540 cases. Again, the LP-M Chart outperforms the CL chart in of 469 them, almost an 87% of them. The performance is more or less similar for downward shifts in the scale parameter (δ=0.5), but for $\delta \geq 1$ and especially $\delta \geq 1.25$, the LP-M chart takes over in 89% and 99% of the cases respectively.

To sum-up, Tables 5.7 to 5.9 show that the LP-M chart has an overall better performance than the CL chart. However, further analysis of the LP-M performance can be explained by the reference sample size “m” and the CUSUM reference value “k”, as done in the next sections.

Table 5.7. LP-M chart Normal distribution OOC performance

θ	m=50, n=5			m=100, n=5		
	MW-Mood CUSUM chart			MW-Mood CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
δ = 0.5						
0	275.5 (15235) 10, 14, 20, 32, 105	2952.3 (323.8) 3001, 3001, 3001, 3001, 3001	2963.8 (281.1) 3001, 3001, 3001, 3001, 3001	***40.1 (810.9) 14, 19, 25, 35, 67	2997.1 (89.2) 3001, 3001, 3001, 3001, 3001	2996.8 (92.0) 3001, 3001, 3001, 3001, 3001
0.25	95.4 (8802) 10, 14, 19, 28, 65	2738.1 (745.1) 462, 3001, 3001, 3001, 3001	2735.9 (741.0) 504, 3001, 3001, 3001, 3001	29.6 (107.3) 14, 19, 24, 32, 54	2897.6 (466.1) 2367.9, 3001, 3001, 3001, 3001	2902.0 (453.2) 2480, 3001, 3001, 3001, 3001
0.5	18.5 (369.6) 7, 11, 14, 19, 32	1719.0 (1320.2) 12, 206, 2226.5, 3001, 3001	1790.0 (1292.9) 18, 297, 2586, 3001, 3001	19.9 (7.4) 11, 15, 19, 23, 33	1871.9 (1256.3) 31, 431, 2931, 3001, 3001	1950.0 (1220.3) 52, 572, 3001, 3001, 3001
0.75	9.5 (4.6) 4, 6, 9, 12, 17	490.7 (944.7) 2, 8, 38, 313, 3001	599.5 (1010.6) 2, 15, 77, 548.25, 3001	11.0 (4.3) 5, 8, 11, 14, 19	312.4 (712.5) 3, 10, 35, 172, 2626.01	446.2 (816.8) 3, 22, 85, 371, 3001
1	5.3 (2.6) 2, 3, 5, 7, 10	52.5 (288.6) 1, 2, 4, 11, 125	88.0 (357.4) 1, 2, 7, 27, 321.056	5.7 (2.3) 3, 4, 5, 7, 10	12.9 (87) 1, 2, 4, 8, 29	33.5 (149) 1, 3, 7, 19, 114
1.5	2.2 (0.8) 1, 2, 2, 3, 4	***1.4 (3.8) 1, 1, 1, 2, 3	***1.7 (6.4) 1, 1, 1, 2, 4	2.4 (0.7) 2, 2, 2, 3, 4	***1.3 (0.6) 1, 1, 1, 2, 2	1.4 (0.9) 1, 1, 1, 2, 3
2	1.4 (0.5) 1, 1, 1, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.6 (0.5) 1, 1, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1 (0.1) 1, 1, 1, 1, 1
3	1.0 (0.05) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.05) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ = 1						
0	480.2 (1759.1) 15, 46, 117, 349, 1840	493.3 (1413.7) 9, 55, 168, 460, 1908	***495.7 (1220.5) 10, 59, 177, 480, 1903	***493.9 (1371.1) 26, 71, 165, 430, 1853	499.9 (861.3) 15, 86, 235, 571, 1827	495.8 (842.4) 16, 89, 241, 574, 1764
0.25	223.5 (1062.9) 10, 25, 54, 144, 778	291.9 (896.7) 5, 29, 89, 255, 1148	294.7 (871.6) 5, 32, 95, 263, 1139	161.6 (487) 14, 33, 64, 137, 544	***251.1 (470.6) 7, 40, 109, 273, 940	258.0 (468.8) 7, 43, 117, 285, 944
0.5	38.5 (143.6) 5, 11, 18, 34, 108	81.0 (242.7) 2, 9, 26, 70, 303	89.4 (248.3) 2, 11, 30, 80, 334	29.1 (37.8) 7, 13, 21, 34, 74	***59.9 (113.2) 3, 11, 28, 66, 213	***68.2 (117.1) 3, 12, 33, 77, 243
0.75	11.4 (12.5) 3, 6, 9, 14, 28	20.4 (51.4) 1, 4, 8, 20, 71	24.5 (62.9) 1, 4, 10, 25, 88	11.4 (7.0) 4, 7, 10, 14, 24	***15.4 (23.6) 1, 4, 9, 18, 51	***19.4 (27.9) 1, 5, 11, 24, 65
1	6.0 (3.8) 2, 4, 5, 7, 13	6.6 (11) 1, 2, 4, 7, 21	8.3 (13.8) 1, 2, 4, 9, 28	6.3 (3.2) 2, 4, 6, 8, 12	***5.5 (5.9) 1, 2, 4, 7, 16	***7.1 (8.6) 1, 2, 4, 9, 22
1.5	2.8 (1.2) 1, 2, 3, 3, 5	1.9 (1.4) 1, 1, 2, 2, 4	2.1 (2.0) 1, 1, 1, 3, 6	3.0 (1.2) 1, 2, 3, 4, 5	1.8 (1.1) 1, 1, 2, 2, 4	2.0 (1.5) 1, 1, 1, 2, 5
2	1.8 (0.7) 1, 1, 2, 2, 3	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	1.9 (0.6) 1, 2, 2, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
3	1.1 (0.3) 1, 1, 1, 1, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.05) 1, 1, 1, 1, 1	1.2 (0.4) 1, 1, 1, 1, 2	1 (0.05) 1, 1, 1, 1, 1	1.0 (0.05) 1, 1, 1, 1, 1
δ = 1.25						
0	42.9 (85.5) 6, 13, 23, 44, 131	62.5 (109.9) 3, 12, 30, 71, 222	69.7 (115.4) 3, 13, 35, 81, 246	37.7 (39.0) 8, 17, 27, 45, 99	62.9 (83.6) 3, 15, 36, 79, 210	72.7 (94.4) 3, 17, 43, 93, 242
0.25	30.8 (50.4) 5, 11, 19, 33, 90	47.5 (93.5) 2, 9, 23, 53, 170	52.8 (88.5) 2, 10, 26, 61, 189	28.3 (26.0) 7, 14, 21, 34, 70	44.6 (60.2) 2, 10, 25, 55, 154	51.9 (67.8) 3, 12, 30, 65, 174.056
0.5	16.5 (48.8) 4, 7, 12, 19, 42	23.2 (41.7) 1, 5, 11, 26, 80	26.8 (44.7) 1, 5, 14, 31, 94	16.0 (11.2) 5, 9, 13, 20, 36	20.6 (26.5) 2, 5, 12, 26, 68	24.9 (31.8) 2, 6, 15, 31, 81
0.75	9.0 (7.1) 3, 5, 7, 11, 21	10.4 (15.0) 1, 3, 6, 12, 33	12.5 (18.8) 1, 3, 7, 15, 41	9.3 (5.4) 3, 6, 8, 12, 19	9.3 (10.5) 1, 3, 6, 12, 28	11.6 (13.7) 1, 3, 7, 15, 37
1	5.6 (3.4) 2, 3, 5, 7, 12	5.2 (6.0) 1, 2, 3, 6, 16	6.3 (7.8) 1, 2, 4, 8, 20	6.0 (3.1) 2, 4, 5, 8, 12	4.8 (4.5) 1, 2, 3, 6, 13	5.8 (6.0) 1, 2, 4, 7, 17
1.5	3.0 (1.4) 1, 2, 3, 4, 5	2.1 (1.5) 1, 1, 2, 3, 5	2.3 (2.0) 1, 1, 2, 3, 6	3.2 (1.4) 1, 2, 3, 4, 6	2.0 (1.3) 1, 1, 2, 3, 5	2.2 (1.7) 1, 1, 2, 3, 6
2	2.0 (0.8) 1, 1, 2, 2, 3	1.3 (0.6) 1, 1, 1, 2, 3	1.4 (0.7) 1, 1, 1, 2, 3	2.1 (0.8) 1, 2, 2, 3, 3	1.3 (0.6) 1, 1, 1, 2, 2	1.3 (0.7) 1, 1, 1, 2, 3
3	1.2 (0.4) 1, 1, 1, 1, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.3 (0.5) 1, 1, 1, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1

δ = 1.5						
0	14.0 (11.9) 4, 7, 11, 17, 34	18.5 (25.2) 1, 5, 11, 23, 61	22.0 (29.4) 1, 5, 13, 27, 72	14.7 (8.9) 5, 9, 13, 18, 31	18.2 (20.6) 2, 5, 12, 24, 57	22.9 (26.0) 2, 6, 15, 30, 72
0.25	12.5 (9.8) 3, 6, 10, 15, 30	15.9 (20.8) 1, 4, 9, 20, 52	18.8 (24.8) 1, 5, 11, 23, 62	13.2 (8.0) 4, 8, 11, 17, 28	15.4 (17.3) 1, 5, 10, 20, 48	19.4 (22.1) 1, 5, 12, 25, 62
0.5	9.6 (6.8) 3, 5, 8, 12, 22	10.8 (13.8) 1, 3, 7, 13, 34	12.9 (16.2) 1, 3, 8, 16, 42	10.2 (5.7) 3, 6, 9, 13, 21	10.2 (11.0) 1, 3, 7, 13, 31	12.8 (14.2) 1, 4, 8, 17, 40
0.75	7.0 (4.4) 2, 4, 6, 9, 15	6.8 (7.8) 1, 2, 4, 8, 21	8.1 (9.4) 1, 2, 5, 10, 25	7.5 (4.0) 3, 5, 7, 9, 15	6.3 (6.2) 1, 2, 4, 8, 18	7.7 (8.0) 1, 2, 5, 10, 23
1	5.1 (2.9) 2, 3, 4, 6, 11	4.3 (4.3) 1, 2, 3, 5, 12	5.0 (5.3) 1, 2, 3, 6, 15	5.5 (2.7) 2, 4, 5, 7, 11	4.1 (3.6) 1, 2, 3, 5, 11	4.8 (4.7) 1, 2, 3, 6, 14
1.5	3.1 (1.5) 1, 2, 3, 4, 6	2.2 (1.6) 1, 1, 2, 3, 5	2.4 (2.0) 1, 1, 2, 3, 6	3.3 (1.4) 1, 2, 3, 4, 6	2.1 (1.4) 1, 1, 2, 3, 5	2.3 (1.8) 1, 1, 2, 3, 6
2	2.1 (0.9) 1, 2, 2, 3, 4	1.5 (0.8) 1, 1, 1, 2, 3	1.5 (0.9) 1, 1, 1, 2, 3	2.3 (0.9) 1, 2, 2, 3, 4	1.4 (0.7) 1, 1, 1, 2, 3	1.5 (0.8) 1, 1, 1, 2, 3
3	1.4 (0.5) 1, 1, 1, 2, 2	1.1 (0.2) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 2	1.5 (0.5) 1, 1, 1, 2, 2	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1
δ = 1.75						
0	8.3 (5.2) 3, 5, 7, 10, 18	8.6 (9.6) 1, 3, 6, 11, 26	10.6 (12.0) 1, 3, 7, 14, 33	9.0 (4.7) 3, 6, 8, 11, 18	8.5 (8.4) 1, 3, 6, 11, 25	10.9 (11.4) 1, 3, 7, 14, 33
0.25	7.8 (4.8) 3, 5, 7, 10, 17	7.9 (8.7) 1, 3, 5, 10, 24	9.7 (11.0) 1, 3, 6, 12, 30	8.5 (4.4) 3, 5, 8, 11, 17	7.8 (7.6) 1, 3, 5, 10, 22	9.9 (10.3) 1, 3, 7, 13, 30
0.5	6.8 (4.0) 2, 4, 6, 9, 14	6.4 (6.8) 1, 2, 4, 8, 19	7.8 (8.7) 1, 2, 5, 10, 24	7.4 (3.8) 3, 5, 7, 9, 14	6.3 (6.0) 1, 2, 4, 8, 18	7.7 (7.8) 1, 2, 5, 10, 23
0.75	5.6 (3.1) 2, 3, 5, 7, 11	4.8 (4.7) 1, 2, 3, 6, 14	5.8 (6.0) 1, 2, 4, 7, 17	6.1 (3.0) 2, 4, 6, 8, 12	4.7 (4.1) 1, 2, 3, 6, 13	5.7 (5.5) 1, 2, 4, 8, 16
1	4.6 (2.4) 2, 3, 4, 6, 9	3.6 (3.3) 1, 2, 3, 5, 10	4.1 (4.0) 1, 1, 3, 5, 12	5.0 (2.3) 2, 3, 5, 6, 9	3.6 (2.9) 1, 2, 3, 5, 9	4.1 (3.7) 1, 2, 3, 5, 11
1.5	3.1 (1.5) 1, 2, 3, 4, 6	2.2 (1.6) 1, 1, 2, 3, 5	2.4 (2.0) 1, 1, 2, 3, 6	3.4 (1.5) 1, 2, 3, 4, 6	2.2 (1.4) 1, 1, 2, 3, 5	2.4 (1.8) 1, 1, 2, 3, 6
2	2.3 (1.0) 1, 2, 2, 3, 4	1.5 (0.86) 1, 1, 1, 2, 3	1.6 (1.0) 1, 1, 1, 2, 4	2.4 (1.0) 1, 2, 2, 3, 4	1.5 (0.8) 1, 1, 1, 2, 3	1.6 (1.0) 1, 1, 1, 2, 3
3	1.5 (0.6) 1, 1, 1, 2, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.6 (0.6) 1, 1, 2, 2, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
δ = 2						
0	5.9 (3.2) 2, 4, 5, 7, 12	5.3 (5.1) 1, 2, 4, 7, 15	6.4 (6.6) 1, 2, 4, 8, 19	6.6 (3.1) 3, 4, 6, 8, 12	5.2 (4.6) 1, 2, 4, 7, 14	6.6 (6.4) 1, 2, 5, 9, 19
0.25	5.8 (3.1) 2, 4, 5, 7, 12	5.1 (4.9) 1, 2, 4, 6, 14	6.1 (6.2) 1, 2, 4, 8, 18	6.4 (3.0) 3, 4, 6, 8, 12	5.0 (4.4) 1, 2, 4, 7, 14	6.2 (6.0) 1, 2, 4, 8, 18
0.5	5.3 (2.8) 2, 3, 5, 7, 11	4.5 (4.2) 1, 2, 3, 6, 13	5.3 (5.3) 1, 2, 4, 7, 16	5.8 (2.7) 2, 4, 5, 7, 11	4.4 (3.7) 1, 2, 3, 6, 12	5.4 (5.1) 1, 2, 4, 7, 15
0.75	4.7 (2.4) 2, 3, 4, 6, 9	3.8 (3.3) 1, 2, 3, 5, 10	4.4 (4.2) 1, 2, 3, 6, 12	5.2 (2.4) 2, 3, 5, 6, 10	3.7 (3.0) 1, 2, 3, 5, 9	4.4 (4.0) 1, 2, 3, 6, 12
1	4.1 (2.0) 2, 3, 4, 5, 8	3.1 (2.6) 1, 1, 2, 4, 8	3.5 (3.3) 1, 1, 2, 5, 10	4.5 (2.0) 2, 3, 4, 6, 8	3.1 (2.3) 1, 1, 2, 4, 8	3.5 (3.1) 1, 1, 3, 5, 10
1.5	3.0 (1.4) 1, 2, 3, 4, 6	2.1 (1.5) 1, 1, 2, 3, 5	2.3 (1.8) 1, 1, 2, 3, 6	3.3 (1.4) 1, 2, 3, 4, 6	2.1 (1.4) 1, 1, 2, 3, 5	2.3(1.8) 1, 1, 2, 3, 6
2	2.3 (1.0) 1, 2, 2, 3, 4	1.6 (0.9) 1, 1, 1, 2, 3	1.7 (1.1) 1, 1, 1, 2, 4	2.5 (1.0) 1, 2, 2, 3, 4	1.6 (0.9) 1, 1, 1, 2, 3	1.7 (1.0) 1, 1, 1, 2, 4
3	1.6 (0.6) 1, 1, 2, 2, 3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	1.7 (0.6) 1, 1, 2, 2, 3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2

Table 5.8. LP-M chart Cauchy distribution OOC performance

θ	m=50, n=5			m=100, n=5		
	MW-SR CUSUM chart			MW-SR CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
δ = 0.5						
0	167.3 (732.0) 17, 32, 57, 120, 526	1946.6 (1140.0) 108, 778, 2461, 3001, 3001	1953.0 (1140.7) 108, 781, 2503.5, 3001, 3001	147.0 (380.8) 27, 46, 75, 135, 433	2266.6 (1023.8) 226, 1419, 3001, 3001, 3001	2236.5 (1033.5) 217, 1351, 3001, 3001, 3001
0.25	121.2 (478.4) 16, 30, 49, 95, 366	1859.2 (1168.6) 80, 640.75, 2180, 3001, 3001	1851.4 (1172.4) 77.95003, 619, 2153, 3001, 3001	108.0 (197.1) 25, 43, 67, 111, 293	2161.9 (1070.4) 172, 1162, 3001, 3001, 3001	2128.4 (1083.2) 166, 1089, 3001, 3001, 3001
0.5	57.5 (130.3) 12, 23, 35, 59, 154	1619.1 (1224.6) 33, 346, 1482, 3001, 3001	1627.8 (1212.9) 37, 375, 1506, 3001, 3001	62.7 (57.9) 20, 34, 49, 73, 146	1855.2 (1170.1) 80, 628, 2166, 3001, 3001	1844.5 (1160.3) 87, 647, 2092, 3001, 3001
0.75	31.2 (39.4) 8, 16, 24, 36, 75	1286.5 (1236.5) 10, 127, 733, 3001, 3001	1331.9 (1226.0) 15, 168, 835.5, 3001, 3001	36.8 (22.8) 12, 22, 32, 45, 78	***1408.5 (1210.9) 24, 233, 1004, 3001, 3001	***1445.3 (1194.2) 34, 289, 1076, 3001, 3001
1	19.1 (13.6) 5, 10, 16, 24, 44	953.7 (1177.9) 4, 34.75, 273, 1873.25, 3001	1027.3 (1176.7) 5, 66, 385, 2087.25, 3001	22.9 (13.2) 8, 14, 20, 29, 47	921.2 (1119.7) 7, 57, 319, 1585, 3001	1039.7 (1127.2) 13, 113, 486, 1908, 3001
1.5	9.6 (6.3) 3, 5, 8, 12, 22	***429.9 (897.9) 1, 4, 19, 216, 3001	551.3 (958.3) 1, 9, 65, 509, 3001	10.5 (5.8) 4, 6, 9, 13, 22	256.5 (666.8) 2, 5, 16, 94, 2055	430.3 (799.4) 2, 14, 71, 370, 3001
2	5.9 (3.7) 2, 3, 5, 7, 13	164.9 (584.3) 1, 2, 4, 16, 1101	287.4 (718.7) 1, 2, 11, 110, 2675	6.2 (3.1) 3, 4, 5, 7, 12	52.9 (297.6) 1, 2, 4, 9, 104	152.1 (471.5) 1, 3, 10, 56, 828
3	3.3 (1.8) 2, 2, 3, 4, 7	24.5 (225.0) 1, 1, 2, 3, 13	82.8 (393.6) 1, 1, 2, 6, 284	3.4 (1.3) 2, 2, 3, 4, 6	3.2 (38.8) 1, 1, 2, 2, 5	18.7 (153.7) 1, 1, 2, 3, 28
δ = 1						
0	502.6 (2008.1) 15, 46, 118, 354, 1875	***501.4 (1458.9) 9, 57, 168, 458, 1893	***496.0 (1283.7) 10, 59, 172, 469, 1892	***507.2 (1696.1) 26, 70, 162, 429, 1861	506.0 (884.1) 15, 87, 239, 578, 1819	498.4 (821.7) 15, 89, 242, 582, 1798
0.25	403.8 (1592.2) 13, 37, 93, 278, 1566.05	473.0 (1421.3) 8, 49, 154, 435, 1796	***477.9 (1182.2) 8, 54, 162, 448, 1827	381.1 (1138.7) 21, 54, 123, 320, 1423	473.1 (878.0) 13, 76, 212, 527, 1749	***473.7 (793.8) 14, 82, 222, 543, 1737
0.5	217.6 (935.2) 9, 23, 53, 148, 825.056	***421.1 (1209.5) 6, 36, 119, 363, 1677	***437.0 (1181.4) 6, 40, 130, 393, 1725	173.8 (476.4) 14, 32, 64, 146, 627	389.0 (773.1) 9, 53, 153, 413, 1489	395.2 (735.6) 10, 59, 171, 435, 1479
0.75	92.7 (349.1) 6, 15, 29, 68, 332	361.0 (1392.2) 3, 21, 77, 270, 1444	370.9 (1212.0) 4, 26, 91, 304, 1487	67.1 (154.9) 9, 19, 33, 63, 212	279.1 (613.0) 5, 29, 91, 274, 1143	306.4 (650.6) 6, 37, 112, 312, 1203
1	40.0 (113.6) 5, 10, 17, 35, 130	283.4 (1435.7) 2, 12, 43, 175, 1151	304.2 (1105.0) 3, 16, 59, 214, 1213	30.9 (52.3) 7, 12, 20, 34, 84	185.5 (497.7) 3, 15, 47, 153, 793	220.0 (519.9) 4, 22, 69, 206, 886
1.5	12.8 (15.8) 3, 5, 9, 15, 35	140.9 (753.8) 1, 4, 12, 51, 560	191.5 (984.1) 1, 6, 20, 87, 760	12.0 (9.2) 4, 7, 10, 14, 28	59.3 (237.2) 2, 5, 12, 35, 225	98.2 (356.9) 2, 7, 22, 70, 395
2	7.1 (6.0) 2, 4, 5, 8, 17	76.7 (707.2) 1, 2, 5, 14, 183	111.0 (858.7) 1, 2, 7, 32, 381	7.0 (4.0) 3, 4, 6, 8, 14	17.8 (110.1) 1, 2, 4, 10, 48	39.0 (180.4) 1, 3, 7, 22, 144
3	3.7 (2.2) 2, 2, 3, 4, 8	15.0 (296.2) 1, 1, 2, 3, 14	37.9 (467.5) 1, 1, 2, 5, 69	3.8 (1.6) 2, 3, 3, 4, 7	3.1 (35.9) 1, 1, 2, 3, 6	6.8 (38.6) 1, 1, 2, 4, 18
δ = 1.25						
0	234.0 (1400.4) 9, 23, 50, 134, 771	209.4 (531.3) 5, 26, 75, 199, 791	215.4 (496.6) 5, 29, 81, 214, 812	157.3 (556.8) 14, 32, 61, 131, 51	200.3 (336.0) 7, 36, 97, 232, 720	212.3 (332.0) 7, 41, 107, 249, 761
0.25	196.9 (968.7) 9, 21, 44, 117, 680	200.3 (744.4) 4, 24, 68, 186, 756	205.6 (471.1) 5, 27, 77, 204, 781	131.4 (356.0) 13, 29, 53.5, 112, 444	191.8 (335.2) 6, 33, 90, 220, 700	205.8 (337.8) 7, 39, 103, 241, 739
0.5	139.1 (1217.6) 7, 16, 32, 79, 446	181.8 (561.0) 3, 18, 56, 163, 708	193.3 (499.8) 4, 21, 64, 178, 757	85.6 (325.6) 10, 21, 38, 74, 264	160.7 (297.9) 5, 25, 70, 176, 605	176.5 (293.4) 5, 30, 83, 201, 652
0.75	70.5 (356.4) 5, 12, 22, 48, 225	159.0 (568.2) 3, 13, 40, 128.25, 632	166.1 (450.5) 3, 16, 49, 147, 667	45.7 (122.5) 8, 15, 25, 45, 129	***122.5 (264.7) 3, 16, 47, 124, 482	***143.8 (271.7) 4, 21, 59, 154, 550
1	35.9 (112.5) 4, 9, 15, 29, 113	124.2 (457.0) 2, 8, 26, 88, 490	137.3 (432.3) 2, 11, 34, 109, 555	25.6 (46.0) 6, 11, 17, 28, 67	86.2 (258.3) 2, 10, 28, 80, 340	108.8 (231.6) 3, 14, 40, 110, 427
1.5	13.0 (21.9) 3, 5, 8, 14, 36	70.6 (391.8) 1, 4, 10, 33, 262	92.2 (398.6) 1, 5, 15, 54, 359	11.7 (9.3) 4, 6, 9, 14, 27	35.0 (124.8) 1, 4, 10, 26, 131	53.9 (150.4) 1, 6, 16, 46, 213
2	7.2 (6.8) 2, 4, 5, 8, 18	40.54 (312.5) 1, 2, 4, 12, 113	56.7 (327.7) 1, 2, 7, 24, 205	7.1 (4.3) 3, 4, 6, 9, 15	13.2 (63.5) 1, 2, 4, 9, 39	25.7 (99.6) 1, 3, 7, 18, 94
3	3.9 (2.4) 2, 2, 3, 5, 8	10.9 (229.8) 1, 1, 2, 4, 14	21.5 (335.2) 1, 1, 2, 5, 47	3.9 (1.7) 2, 3, 4, 5, 7	3.0 (10.0) 1, 1, 2, 3, 7	6.1 (58.9) 1, 1, 2, 4, 16

δ = 1.5						
0	96.3 (668.3) 6, 15, 27, 60, 286	104.4 (254.3) 3, 14, 39, 103, 393	112.6 (249.8) 3, 16, 45, 114, 417	56.6 (120.4) 9, 19, 32, 57, 166	98.2 (162.3) 4, 19, 48, 113, 352	112.3 (168.6) 4, 23, 59, 135, 394
0.25	88.1 (610.8) 6, 14, 26, 55, 258	101.5 (263.1) 3, 13, 37, 99, 375	110.4 (310.2) 3, 15, 43, 110, 409	53.9 (148.7) 9, 18, 30, 53, 151	94.0 (161.9) 4, 17, 45, 107, 337	106.8 (169.3) 4, 21, 55, 126, 378
0.5	64.7 (335.4) 5, 12, 21, 44, 193	94.6 (247.2) 2, 11, 32, 88, 359	103.7 (272.0) 3, 13, 38, 102, 389	40.6 (121.6) 8, 15, 24, 42, 110	81.5 (142.8) 3, 14, 37, 91, 297	95.2 (157.9) 3, 17, 46, 110, 342
0.75	44.2 (224.6) 5, 10, 17, 33, 127	80.0 (214.6) 2, 8, 24, 70, 318	92.7 (264.2) 2, 10, 30, 85, 361	28.5 (50.3) 6, 12, 19, 31, 74	64.2 (127.3) 3, 10, 28, 68, 242	***80.7 (142.8) 3, 13, 36, 89, 299
1	27.6 (122.2) 4, 8, 13, 23, 75	65.1 (289.0) 2, 6, 17, 52, 257	78.4 (227.6) 2, 8, 23, 67.25, 315	19.8 (22.4) 5, 9, 14, 23, 50	48.1 (104.8) 2, 7, 18, 47, 183	64.7 (125.7) 2, 10, 27, 69, 246
1.5	12.3 (22.5) 3, 5, 8, 13, 32	40.8 (192.5) 1, 3, 8, 24, 154	***54.3 (218.6) 1, 4, 12, 37, 214	10.9 (8.5) 4, 6, 9, 13, 24	22.9 (68.8) 1, 4, 8, 20, 81	35.3 (92.3) 1, 5, 13, 33, 134
2	7.2 (7.6) 2, 4, 5, 8, 18	24.0 (166.5) 1, 2, 4, 11, 74	35.2 (176.9) 1, 2, 6, 19, 128	7.0 (4.1) 3, 4, 6, 8, 14	9.8 (29.3) 1, 2, 4, 9, 31	18.2 (57.7) 1, 3, 6, 15, 66
3	3.9 (2.5) 2, 2, 3, 5, 8	8.0 (158.7) 1, 1, 2, 4, 14	14.8 (124.3) 1, 1, 2, 5, 37	4.0 (1.8) 2, 3, 4, 5, 7	3.0 (5.1) 1, 1, 2, 3, 7	5.2 (18.3) 1, 1, 2, 4, 15
δ = 1.75						
0	43.3 (247.9) 5, 10, 18, 34, 121	59.3 (142.4) 2, 9, 24, 59, 221	68.7 (155.0) 2, 11, 29, 72, 251	30.0 (52.0) 7, 13, 21, 34, 76	54.7 (88.3) 3, 11, 28, 65, 192	66.1 (99.2) 3, 14, 35, 79, 227
0.25	44.1 (339.7) 5, 10, 17, 32, 112	58.9 (144.5) 2, 8, 23, 58, 216	67.1 (146.8) 2, 10, 27, 69, 248	28.2 (36.4) 7, 13, 20, 32, 72	52.8 (84.5) 3, 11, 27, 61, 186	64.8 (97.1) 3, 13, 34, 77, 226
0.5	34.1 (137.2) 4, 9, 15, 28, 95	53.4 (123.7) 2, 7, 20, 52, 202	61.6 (137.5) 2, 9, 25, 62, 232	24.4 (36.1) 6, 11, 18, 28, 61	46.8 (77.5) 2, 9, 23, 53, 170	57.7 (89.1) 2, 12, 30, 69, 203
0.75	27.0 (151.9) 4, 8, 13, 23, 72	47.2 (138.7) 2, 6, 16, 43, 181	56.9 (138.9) 2, 8, 21, 55, 216	20.1 (44.5) 5, 10, 15, 23, 48	39.1 (71.8) 2, 7, 18, 43, 142	49.7 (81.2) 2, 9, 24, 58, 177
1	20.0 (119.7) 3, 7, 11, 18, 52	40.1 (111.0) 1, 5, 12, 34, 157	49.2 (126.0) 1, 6, 16, 45, 195	15.7 (15.7) 5, 8, 12, 19, 37	30.4 (59.5) 2, 6, 13, 32, 112	40.9 (75.6) 2, 7, 19, 45, 150
1.5	11.0 (20.6) 3, 5, 7, 12, 28	26.8 (110.7) 1, 3, 7, 19, 98	34.5 (102.8) 1, 4, 10, 28, 135	9.9 (7.3) 3, 6, 8, 12, 22	16.4 (39.0) 1, 3, 7, 16, 57	24.7 (54.2) 1, 4, 10, 25, 91
2	7.0 (7.5) 2, 4, 5, 8, 17	***16.0 (69.2) 1, 2, 4, 10, 54	24.2 (99.8) 1, 2, 6, 15, 89	6.8 (3.8) 3, 4, 6, 8, 14	8.3 (19.6) 1, 2, 4, 8, 26	13.9 (33.0) 1, 2, 6, 13, 49
3	4.0 (2.5) 2, 2, 3, 5, 8	***5.6 (36.2) 1, 1, 2, 4, 13	10.2 (58.7) 1, 1, 2, 5, 31	4.1 (1.8) 2, 3, 4, 5, 7	***3.0 (3.7) 1, 1, 2, 3, 7	4.8 (17.6) 1, 1, 2, 5, 14
δ = 2						
0	25.4 (310.0) 4, 8, 13, 23, 63	37.1 (87.7) 2, 6, 15, 38, 133	43.8 (87.6) 2, 7, 19, 47, 160	19.7 (19.5) 6, 10, 15, 23, 47	33.4 (50.6) 2, 7, 18, 39, 116	42.5 (60.8) 2, 9, 23, 52, 145
0.25	24.1 (102.7) 4, 8, 13, 22, 62	36.7 (85.9) 2, 6, 15, 37, 135	43.8 (91.1) 2, 7, 19, 46, 158	19.0 (20.8) 5, 10, 15, 23, 45	32.1 (48.6) 2, 7, 17, 38, 112	41.7 (59.8) 2, 9, 23, 50, 141
0.5	21.9 (109.5) 4, 7, 12, 20, 54	33.7 (84.9) 2, 5, 13, 33, 123	41.3 (91.7) 2, 7, 17, 43, 149	17.1 (14.2) 5, 9, 14, 21, 40	29.3 (45.5) 2, 6, 15, 34, 103	38.6 (59.0) 2, 8, 21, 46, 133
0.75	18.3 (84.6) 3, 7, 11, 18, 45	31.1 (83.0) 1, 5, 12, 30, 115	37.6 (83.7) 1, 6, 15, 38, 141	15.1 (13.4) 5, 8, 12, 18, 34	25.0 (41.7) 2, 5, 13, 28, 86	34.2 (53.7) 2, 7, 17, 40, 121
1	14.9 (51.0) 3, 6, 9, 15, 37	27.0 (73.1) 1, 4, 9, 24, 99	33.8 (81.3) 1, 5, 12, 33, 130	12.7 (9.9) 4, 7, 10, 15, 28	20.4 (36.7) 2, 4, 10, 22, 71	28.6 (46.1) 1, 6, 14, 33, 102
1.5	10.1 (108.7) 3, 4, 7, 11, 24	18.9 (62.3) 1, 3, 6, 15, 70	25.7 (82.4) 1, 3, 8, 22, 96	9.0 (6.4) 3, 5, 8, 11, 19	***12.2 (24.3) 1, 3, 6, 13, 40	18.9 (37.2) 1, 4, 9, 20, 67
2	6.7 (7.4) 2, 4, 5, 8, 15	11.8 (43.0) 1, 2, 4, 9, 40	17.8 (62.2) 1, 2, 5, 13, 65	6.6 (3.7) 3, 4, 6, 8, 13	7.1 (14.2) 1, 2, 4, 7, 22	11.6 (27.1) 1, 2, 5, 12, 40
3	4.1 (2.7) 2, 3, 3, 5, 8	5.0 (26.1) 1, 1, 2, 4, 12	8.8 (60.6) 1, 1, 2, 5, 26	4.1 (1.8) 2, 3, 4, 5, 8	***3.0 (3.6) 1, 1, 2, 3, 7	4.4 (9.1) 1, 1, 2, 5, 13

Table 5.9. LP-M chart Lognormal distribution OOC performance

θ	m=50, n=5			m=100, n=5		
	MW-SR CUSUM chart			MW-SR CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
δ = 0.5						
-3	1.0 (0.05) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
-2	1.4 (0.5) 1, 1, 1, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.6 (0.5) 1, 1, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
-1.5	2.2 (0.8) 1, 2, 2, 3, 4	1.4 (1.1) 1, 1, 1, 2, 3	1.7 (3.7) 1, 1, 1, 2, 4	2.4 (0.7) 2, 2, 2, 3, 4	1.3 (0.6) 1, 1, 1, 2, 2	1.4 (0.9) 1, 1, 1, 2, 3
-1	5.3 (2.6) 2, 3, 5, 7, 10	***53.2 (291.1) 1, 2, 4, 11, 121	88.9 (363.6) 1, 2, 7, 26, 328	5.7 (2.3) 3, 4, 5, 7, 10	12.1 (75.2) 1, 2, 4, 8, 30	33.6 (151.8) 1, 3, 7, 19, 112
-0.75	9.5 (4.6) 4, 6, 9, 12, 17	495.1 (952.6) 2, 8, 36, 312, 3001	592.5 (1000.2) 2, 15, 79, 543, 3001	11.0 (4.3) 5, 8, 11, 14, 19	317.7 (722.1) 3, 10, 34, 175, 2763.01	441.5 (807.7) 3, 21, 85, 372, 3001
-0.5	***18.6 (225.1) 7, 11, 14, 19, 32	1715.2 (1318.5) 12, 202, 2173, 3001, 3001	1779.2 (1296.0) 18, 289, 2526, 3001, 3001	19.9 (7.3) 11, 15, 19, 23, 32	1880.3 (1251.9) 32, 448, 2939.5, 3001, 3001	1944.8 (1219.4) 51, 572, 3001, 3001, 3001
-0.25	68.9 (3214.5) 10, 14, 19, 28, 64	2724.2 (763.5) 413, 3001, 3001, 3001, 3001	2739.2 (741.2) 486, 3001, 3001, 3001, 3001	***28.8 (39.4) 14, 19, 24, 32, 54	2900.9 (457.7) 2469, 3001, 3001, 3001, 3001	2895.2 (468.6) 2271.7, 3001, 3001, 3001, 3001
0	286.1 (18511.5) 9, 14, 20, 32, 101	2951.9 (326.7) 3001, 3001, 3001, 3001, 3001	2965.3 (277.2) 3001, 3001, 3001, 3001, 3001	***38.2 (705.3) 14, 19, 25, 35, 68	2997.1 (88.8) 3001, 3001, 3001, 3001, 3001	2997.7 (79.9) 3001, 3001, 3001, 3001, 3001
0.25	51.3 (1395.8) 10, 14, 19, 28, 65	2732.4 (753.5) 442.95, 3001, 3001, 3001, 3001	2741.0 (738.8) 497, 3001, 3001, 3001, 3001	28.6 (30.8) 14, 19, 24, 32, 54	2902.5 (451.7) 2469.8, 3001, 3001, 3001, 3001	2899.4 (458.9) 2388, 3001, 3001, 3001, 3001
0.5	16.9 (22.1) 7, 11, 14, 19, 31	1720.6 (1319.2) 12, 202, 2235, 3001, 3001	1779.1 (1296.0) 18, 286, 2517, 3001, 3001	19.9 (7.3) 11, 15, 19, 23, 32	1873.5 (1256.5) 32, 437, 2958, 3001, 3001	1940.4 (1220.6) 50, 568, 3001, 3001, 3001
0.75	9.4 (4.6) 4, 6, 9, 12, 17	489.5 (943.5) 2, 8, 37, 309.25, 3001	592.7 (1003.6) 2, 15, 78, 533, 3001	11.0 (4.3) 5, 8, 11, 14, 19	317.6 (721.6) 3, 10, 34, 174, 2718.05	446.0 (811.9) 3, 22, 86, 379, 3001
1	5.3 (2.6) 2, 3, 5, 7, 10	49.6 (278.9) 1, 2, 4, 10, 116	91.4 (372.5) 1, 2, 7, 26, 337	5.7 (2.3) 3, 4, 5, 7, 10	12.8 (85.5) 1, 2, 4, 8, 30	34.0 (150.6) 1, 3, 7, 19, 114
1.5	2.2 (0.8) 1, 2, 2, 3, 4	***1.4 (3.9) 1, 1, 1, 2, 3	***1.8 (9.4) 1, 1, 1, 2, 4	2.4 (0.7) 2, 2, 2, 3, 4	***1.3 (0.6) 1, 1, 1, 2, 2	***1.4 (0.9) 1, 1, 1, 2, 3
2	1.4 (0.5) 1, 1, 1, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.6 (0.5) 1, 1, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
3	1.0 (0.05) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ = 1						
-3	1.1 (0.3) 1, 1, 1, 1, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.2 (0.4) 1, 1, 1, 1, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
-2	1.8 (0.7) 1, 1, 2, 2, 3	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	1.9 (0.6) 1, 2, 2, 2, 3	***1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
-1.5	2.8 (1.2) 1, 2, 3, 3, 5	1.9 (1.4) 1, 1, 2, 2, 4	2.1 (1.9) 1, 1, 1, 2, 5	3.0 (1.2) 1, 2, 3, 4, 5	1.8 (1.1) 1, 1, 2, 2, 4	2.0 (1.5) 1, 1, 1, 2, 5
-1	6.0 (3.8) 2, 3, 5, 7, 13	6.6 (12.6) 1, 2, 4, 7, 21	8.4 (14.4) 1, 2, 4, 9, 28	6.3 (3.2) 2, 4, 6, 8, 12	5.4 (5.9) 1, 2, 4, 7, 16	7.1 (8.4) 1, 2, 4, 9, 22
-0.75	11.4 (13.5) 3, 6, 9, 14, 27	20.2 (50.5) 1, 4, 8, 20, 72	24.2 (55.0) 1, 4, 10, 25, 87	11.4 (7.3) 4, 7, 10, 14, 24	15.3 (22.4) 1, 4, 9, 18, 50	19.4 (28.5) 1, 4, 11, 23, 65
-0.5	38.4 (378.6) 5, 11, 18, 34, 105	85.2 (595.0) 2, 9, 26, 71, 306	89.1 (290.0) 2, 11, 30, 80, 330	29.0 (36.4) 7, 13, 21, 34, 73	59.0 (106.3) 3, 11, 28, 66, 211	68.5 (117.9) 3, 13, 33, 77, 247
-0.25	219.3 (1050.1) 10, 25, 54, 142, 792	297.7 (1055.7) 5, 29, 88, 256, 1148.05	300.2 (833.1) 5, 32, 97, 271, 1161.05	161.0 (510.8) 14, 33, 64, 138, 552	250.3 (471.0) 7, 40, 108, 271, 931	253.7 (446.6) 7, 43, 116, 284, 927.056
0	488.3 (2535.4) 15, 46, 118, 343, 1810.05	***497.1 (1391.8) 9, 56, 164, 457, 1878	***520.2 (1919.1) 10, 59, 176, 479, 1933	***503.0 (1862.1) 26, 70, 164, 428.25, 1820	493.9 (861.4) 14, 85.75, 234, 564, 1788	496.5 (826.2) 16, 91, 243, 570, 1779
0.25	***241.8 (1529.8) 10, 25, 54, 142, 806	***288.2 (808.3) 5, 29, 89, 254, 1128	298.1 (849.0) 5, 32, 97, 270, 1146.05	163.4 (525.5) 15, 34, 64, 140, 552.056	248.4 (462.4) 7, 39, 108, 272, 932	258.4 (454.1) 8, 43, 117, 288, 956
0.5	39.6 (285.0) 5, 11, 18, 34, 108	83.0 (356.2) 2, 9, 26, 71, 304	91.2 (299.3) 2, 11, 31, 81, 334	28.9 (33.6) 7, 13, 21, 34, 74	59.0 (106.2) 2, 11, 27, 66, 213	68.8 (118.0) 3, 13, 33, 78, 245
0.75	11.4 (11.6) 3, 6, 9, 14, 28	***20.6 (62.2) 1, 4, 8, 20, 72	***24.2 (54.1) 1, 4, 10, 25, 85	11.5 (7.2) 4, 7, 10, 14, 25	***15.4 (23.1) 1, 4, 8, 18, 52	***19.3 (29.0) 1, 4, 11, 23, 64
1	6.0 (3.8) 2, 3, 5, 7, 13	6.6 (10.8) 1, 2, 4, 7, 21	***8.3 (14.0) 1, 2, 4, 9, 28	6.3 (3.2) 2, 4, 6, 8, 12	5.5 (5.9) 1, 2, 4, 7, 16	***7.1 (8.7) 1, 2, 4, 9, 22
1.5	2.8 (1.2) 1, 2, 3, 3, 5	1.9 (1.4) 1, 1, 1, 2, 4	2.1 (1.9) 1, 1, 1, 3, 6	3.0 (1.2) 1, 2, 3, 4, 5	1.8 (1.1) 1, 1, 2, 2, 4	2.0 (1.5) 1, 1, 2, 2, 5
2	1.8 (0.7) 1, 1, 2, 2, 3	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	1.9 (0.6) 1, 2, 2, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
3	1.1 (0.3) 1, 1, 1, 1, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.2 (0.4) 1, 1, 1, 1, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1

δ = 1.25						
-3	1.2 (0.4) 1, 1, 1, 1, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.3 (0.5) 1, 1, 1, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
-2	2.0 (0.8) 1, 1, 2, 2, 3	1.3 (0.6) 1, 1, 1, 2, 3	1.4 (0.7) 1, 1, 1, 2, 3	2.1 (0.8) 1, 2, 2, 3, 3	***1.3 (0.6) 1, 1, 1, 2, 2	1.3 (0.7) 1, 1, 1, 2, 3
-1.5	3.0 (1.4) 1, 2, 3, 4, 6	2.0 (1.5) 1, 1, 2, 3, 5	2.3 (2.0) 1, 1, 2, 3, 6	3.2 (1.4) 1, 2, 3, 4, 6	2.0 (1.3) 1, 1, 2, 3, 5	2.2 (1.7) 1, 1, 2, 3, 5
-1	5.7 (3.4) 2, 3, 5, 7, 12	5.2 (6.0) 1, 2, 3, 6, 15	6.3 (7.9) 1, 2, 4, 8, 20	6.0 (3.1) 2, 4, 5, 8, 12	4.8 (4.4) 1, 2, 3, 6, 13	5.8 (5.9) 1, 2, 4, 7, 17
-0.75	9.0 (6.8) 3, 5, 7, 11, 21	10.4 (15.2) 1, 3, 6, 12, 33	12.4 (17.9) 1, 3, 7, 15, 41	9.4 (5.4) 3, 6, 8, 12, 19	9.3 (10.4) 1, 3, 6, 12, 28	11.4 (13.1) 1, 3, 7, 15, 36
-0.5	16.2 (19.1) 4, 7, 12, 19, 42	23.2 (38.8) 1, 5, 12, 26, 80	26.9 (42.8) 1, 6, 14, 31, 96	15.9 (10.9) 5, 9, 13, 20, 36	20.6 (26.2) 2, 5, 12, 26, 67	25.1 (31.9) 2, 6, 15, 32, 82
-0.25	31.5 (57.4) 5, 11, 19, 33, 91	46.7 (81.3) 2, 9, 22, 53, 168	52.6 (87.2) 2, 10, 26, 60.25, 189	28.2 (25.2) 7, 14, 22, 34, 70	44.8 (59.9) 3, 10, 25, 56, 150	51.6 (67.5) 3, 12, 30, 65, 172
0	43.4 (103.5) 6, 13, 24, 44, 130	62.9 (108.4) 3, 12, 30, 71, 225	69.2 (115.4) 3, 13, 35, 80, 243	37.6 (41.4) 8, 17, 27, 45, 99	62.8 (83.0) 3, 15, 36, 79, 212	72.3 (91.8) 3, 17, 43, 92, 239
0.25	32.2 (118.7) 5, 11, 19, 33, 90	46.5 (82.9) 2, 9, 22, 53, 166	52.7 (88.6) 2, 10, 26, 60, 188	28.2 (26.0) 7, 14, 22, 34, 71	44.4 (59.3) 2, 10, 25, 55, 151	52.1 (69.1) 3, 12, 30, 65, 175
0.5	16.3 (21.0) 4, 7, 12, 19, 42	23.5 (41.7) 1, 5, 12, 26, 81	26.5 (42.6) 1, 5, 13, 31, 93	16.0 (11.3) 5, 9, 13, 20, 36	20.7 (26.7) 2, 5, 12, 26, 67	24.9 (32.2) 2, 6, 15, 31, 81
0.75	9.0 (6.8) 3, 5, 7, 11, 21	10.4 (15.4) 1, 3, 6, 12, 34	12.4 (18.1) 1, 3, 7, 15, 41	9.4 (5.4) 3, 6, 8, 12, 19	9.3 (10.5) 1, 3, 6, 12, 28	11.4 (13.3) 1, 3, 7, 15, 36
1	5.7 (3.4) 2, 3, 5, 7, 12	5.2 (6.2) 1, 2, 3, 6, 15	6.3 (8.2) 1, 2, 4, 8, 19	6.0 (3.1) 2, 4, 5, 8, 12	4.7 (4.5) 1, 2, 3, 6, 13	5.8 (5.9) 1, 2, 4, 8, 17
1.5	3.0 (1.4) 1, 2, 3, 4, 6	2.1 (1.5) 1, 1, 2, 3, 5	2.3 (2.0) 1, 1, 2, 3, 6	3.2 (1.4) 1, 2, 3, 4, 6	2.0 (1.3) 1, 1, 2, 3, 5	2.2 (1.7) 1, 1, 2, 3, 6
2	2.0 (0.8) 1, 1, 2, 2, 3	1.3 (0.6) 1, 1, 1, 2, 3	1.4 (0.7) 1, 1, 1, 2, 3	2.1 (0.8) 1, 2, 2, 3, 3	1.3 (0.6) 1, 1, 1, 2, 2	1.3 (0.7) 1, 1, 1, 2, 3
3	1.2 (0.4) 1, 1, 1, 1, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.3 (0.5) 1, 1, 1, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
δ = 1.5						
-3	1.4 (0.5) 1, 1, 1, 2, 2	1.1 (0.2) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 1	1.5 (0.5) 1, 1, 1, 2, 2	1.1 (0.2) 1, 1, 1, 1, 2	1.0 (0.2) 1, 1, 1, 1, 1
-2	2.2 (0.9) 1, 2, 2, 3, 4	1.4 (0.7) 1, 1, 1, 2, 3	1.5 (0.9) 1, 1, 1, 2, 3	2.3 (0.9) 1, 2, 2, 3, 4	1.4 (0.7) 1, 1, 1, 2, 3	1.5 (0.8) 1, 1, 1, 2, 3
-1.5	3.1 (1.5) 1, 2, 3, 4, 6	2.2 (1.6) 1, 1, 2, 3, 5	2.4 (2.0) 1, 1, 2, 3, 6	3.3 (1.5) 1, 2, 3, 4, 6	2.2 (1.4) 1, 1, 2, 3, 5	2.3 (1.8) 1, 1, 2, 3, 6
-1	5.1 (2.9) 2, 3, 5, 6, 10	4.3 (4.3) 1, 2, 3, 5, 12	5.0 (5.4) 1, 2, 3, 6, 15	5.5 (2.7) 2, 4, 5, 7, 11	4.1 (3.6) 1, 2, 3, 5, 11	4.9 (4.7) 1, 2, 3, 6, 14
-0.75	7.0 (4.4) 2, 4, 6, 9, 15	6.7 (7.7) 1, 2, 4, 8, 20	8.0 (9.5) 1, 2, 5, 10, 25	7.5 (4.0) 3, 5, 7, 9, 15	6.4 (6.3) 1, 2, 4, 8, 18	7.8 (8.0) 1, 2, 5, 10, 23
-0.5	9.6 (6.9) 3, 5, 8, 12, 22	10.8 (13.9) 1, 3, 7, 13, 35	12.8 (15.8) 1, 3, 8, 16, 42	10.3 (5.8) 3, 6, 9, 13, 21	10.3 (11.1) 1, 3, 7, 13, 31	12.7 (14.0) 1, 4, 8, 17, 39
-0.25	12.6 (10.2) 3, 6, 10, 16, 30	15.9 (21.1) 1, 4, 9, 20, 52	19.0 (24.8) 1, 5, 11, 24, 63	13.2 (8.0) 4, 8, 11, 17, 28	15.3 (17.1) 1, 5, 10, 20, 48	19.1 (21.6) 1, 5, 12, 25, 60
0	14.0 (12.0) 4, 7, 11, 17, 34	18.6 (25.2) 1, 5, 11, 23, 62	22.1 (29.5) 1, 5, 13, 28, 73	14.6 (9.0) 5, 9, 13, 18, 31	18.4 (21.0) 2, 5, 12, 24, 57	22.6 (25.9) 2, 6, 14, 30, 71
0.25	12.6 (10.2) 3, 6, 10, 16, 30	15.8 (20.8) 1, 4, 9, 19, 53	18.7 (24.3) 1, 5, 11, 23, 62	13.2 (7.9) 4, 8, 11, 17, 28	15.3 (16.9) 1, 5, 10, 20, 47	19.4 (22.2) 1, 5, 12, 25, 61
0.5	9.6 (7.0) 3, 5, 8, 12, 22	10.7 (13.5) 1, 3, 6, 13, 34	12.8 (16.1) 1, 3, 8, 16, 41	10.2 (5.7) 3, 6, 9, 13, 21	10.3 (11.0) 1, 3, 7, 13, 31	12.6 (13.9) 1, 4, 8, 17, 39
0.75	7.0 (4.4) 2, 4, 6, 9, 15	6.8 (7.7) 1, 2, 4, 8, 20	8.0 (9.5) 1, 2, 5, 10, 25	7.5 (3.9) 3, 5, 7, 9, 15	6.4 (6.3) 1, 2, 4, 8, 18	7.8 (8.2) 1, 2, 5, 10, 23
1	5.1 (2.9) 2, 3, 4, 6, 10	4.3 (4.3) 1, 2, 3, 5, 12	5.0 (5.3) 1, 2, 3, 6, 15	5.5 (2.7) 2, 4, 5, 7, 11	4.1 (3.6) 1, 2, 3, 5, 11	4.9 (4.7) 1, 2, 3, 6, 14
1.5	3.1 (1.5) 1, 2, 3, 4, 6	2.2 (1.6) 1, 1, 2, 3, 5	2.4 (2.0) 1, 1, 2, 3, 6	3.3 (1.5) 1, 2, 3, 4, 6	2.1 (1.4) 1, 1, 2, 3, 5	2.3 (1.8) 1, 1, 2, 3, 6
2	2.2 (0.9) 1, 2, 2, 3, 4	1.5 (0.8) 1, 1, 1, 2, 3	1.5 (0.9) 1, 1, 1, 2, 3	2.3 (0.9) 1, 2, 2, 3, 4	1.4 (0.7) 1, 1, 1, 2, 3	1.5 (0.8) 1, 1, 1, 2, 3
3	1.4 (0.5) 1, 1, 1, 2, 2	1.1 (0.2) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 1	1.5 (0.5) 1, 1, 1, 2, 2	1.1 (0.2) 1, 1, 1, 1, 2	1.0 (0.2) 1, 1, 1, 1, 1

δ = 1.75						
-3	1.5 (0.6) 1, 1, 1, 2, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.6 (0.6) 1, 1, 2, 2, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
-2	2.3 (1.0) 1, 2, 2, 3, 4	1.5 (0.9) 1, 1, 1, 2, 3	1.6 (1.0) 1, 1, 1, 2, 4	2.5 (1.0) 1, 2, 2, 3, 4	1.5 (0.8) 1, 1, 1, 2, 3	1.6 (1.0) 1, 1, 1, 2, 3
-1.5	3.1 (1.5) 1, 2, 3, 4, 6	2.2 (1.6) 1, 1, 2, 3, 5	2.4 (1.9) 1, 1, 2, 3, 6	3.4 (1.5) 1, 2, 3, 4, 6	2.2 (1.4) 1, 1, 2, 3, 5	2.4 (1.8) 1, 1, 2, 3, 6
-1	4.6 (2.4) 2, 3, 4, 6, 9	3.6 (3.2) 1, 2, 3, 5, 10	4.2 (4.1) 1, 1, 3, 5, 12	5.0 (2.3) 2, 3, 5, 6, 9	3.5 (2.8) 1, 2, 3, 5, 9	4.1 (3.8) 1, 2, 3, 5, 11
-0.75	5.6 (3.2) 2, 3, 5, 7, 12	4.9 (4.8) 1, 2, 3, 6, 14	5.7 (6.0) 1, 2, 4, 7, 17	6.1 (3.0) 2, 4, 6, 8, 12	4.7 (4.2) 1, 2, 3, 6, 13	5.7 (5.5) 1, 2, 4, 8, 17
-0.5	6.8 (4.1) 2, 4, 6, 9, 14	6.5 (6.8) 1, 2, 4, 8, 19	7.7 (8.5) 1, 2, 5, 10, 24	7.4 (3.7) 3, 5, 7, 9, 14	6.3 (5.9) 1, 2, 4, 8, 18	7.8 (7.8) 1, 2, 5, 10, 23
-0.25	7.8 (4.8) 3, 5, 7, 10, 17	8.0 (8.7) 1, 3, 5, 10, 24	9.6 (10.8) 1, 3, 6, 12, 30	8.5 (4.4) 3, 5, 8, 11, 17	7.8 (7.5) 1, 3, 5, 10, 22	10.0 (10.5) 1, 3, 7, 13, 30
0	8.3 (5.2) 3, 5, 7, 10, 18	8.7 (9.7) 1, 3, 6, 11, 26	10.5 (12.1) 1, 3, 7, 14, 33	9.0 (4.7) 3, 6, 8, 11, 18	8.5 (8.5) 1, 3, 6, 11, 25	10.8 (11.4) 1, 3, 7, 14, 33
0.25	7.8 (4.8) 3, 5, 7, 10, 17	8.0 (8.7) 1, 3, 5, 10, 24	9.6 (11.2) 1, 3, 6, 12, 30	8.5 (4.4) 3, 5, 8, 11, 17	7.8 (7.7) 1, 3, 5, 10, 23	9.9 (10.2) 1, 3, 7, 13, 30
0.5	6.8 (4.1) 2, 4, 6, 9, 14	6.4 (6.7) 1, 2, 4, 8, 19	7.8 (8.5) 1, 2, 5, 10, 24	7.4 (3.8) 3, 5, 7, 9, 15	6.3 (5.9) 1, 2, 4, 8, 18	7.8 (7.8) 1, 3, 5, 10, 23
0.75	5.6 (3.2) 2, 3, 5, 7, 12	4.9 (4.8) 1, 2, 3, 6, 14	5.7 (5.9) 1, 2, 4, 7, 17	6.1 (3.0) 2, 4, 6, 8, 12	4.7 (4.1) 1, 2, 4, 6, 13	5.7 (5.5) 1, 2, 4, 7, 16
1	4.6 (2.4) 2, 3, 4, 6, 9	3.6 (3.2) 1, 2, 3, 5, 10	4.2 (4.1) 1, 1, 3, 5, 12	5.0 (2.4) 2, 3, 5, 6, 9	3.5 (2.9) 1, 2, 3, 5, 9	4.1 (3.8) 1, 2, 3, 5, 11
1.5	3.1 (1.5) 1, 2, 3, 4, 6	2.2 (1.6) 1, 1, 2, 3, 5	2.4 (2.0) 1, 1, 2, 3, 6	3.4 (1.5) 1, 2, 3, 4, 6	2.2 (1.4) 1, 1, 2, 3, 5	2.4 (1.8) 1, 1, 2, 3, 6
2	2.3 (1.0) 1, 2, 2, 3, 4	1.5 (0.9) 1, 1, 1, 2, 3	1.6 (1.0) 1, 1, 1, 2, 4	2.5 (1.0) 1, 2, 2, 3, 4	1.5 (0.8) 1, 1, 1, 2, 3	1.6 (1.0) 1, 1, 1, 2, 3
3	1.5 (0.6) 1, 1, 1, 2, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.6 (0.6) 1, 1, 2, 2, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
δ = 2						
-3	1.6 (0.6) 1, 1, 2, 2, 3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	1.7 (0.6) 1, 1, 2, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2
-2	2.4 (1.0) 1, 2, 2, 3, 4	1.6 (0.9) 1, 1, 1, 2, 3	1.7 (1.1) 1, 1, 1, 2, 4	2.5 (1.0) 1, 2, 2, 3, 4	1.6 (0.9) 1, 1, 1, 2, 3	1.7 (1.0) 1, 1, 1, 2, 4
-1.5	3.0 (1.4) 1, 2, 3, 4, 6	2.1 (1.5) 1, 1, 2, 3, 5	2.3 (1.8) 1, 1, 2, 3, 6	3.3 (1.4) 1, 2, 3, 4, 6	2.1 (1.4) 1, 1, 2, 3, 5	2.3 (1.7) 1, 1, 2, 3, 6
-1	4.1 (2.1) 2, 3, 4, 5, 8	3.1 (2.5) 1, 1, 2, 4, 8	3.5 (3.2) 1, 1, 2, 4, 10	4.5 (2.0) 2, 3, 4, 6, 8	3.1 (2.3) 1, 1, 2, 4, 8	3.5 (3.0) 1, 1, 3, 5, 10
-0.75	4.7 (2.5) 2, 3, 4, 6, 9	3.8 (3.3) 1, 2, 3, 5, 10	4.4 (4.2) 1, 2, 3, 6, 12	5.2 (2.4) 2, 3, 5, 6, 10	3.7 (3.0) 1, 2, 3, 5, 10	4.4 (4.0) 1, 2, 3, 6, 12
-0.5	5.3 (2.8) 2, 3, 5, 7, 11	4.5 (4.1) 1, 2, 3, 6, 12	5.3 (5.3) 1, 2, 4, 7, 15	5.8 (2.8) 2, 4, 5, 7, 11	4.4 (3.8) 1, 2, 3, 6, 12	5.4 (5.0) 1, 2, 4, 7, 15
-0.25	5.8 (3.1) 2, 4, 5, 7, 12	5.1 (4.9) 1, 2, 4, 6, 14	6.1 (6.3) 1, 2, 4, 8, 18	6.4 (3.0) 3, 4, 6, 8, 12	5.0 (4.4) 1, 2, 4, 7, 14	6.2 (6.0) 1, 2, 4, 8, 18
0	6.0 (3.3) 2, 4, 5, 7, 12	5.3 (5.1) 1, 2, 4, 7, 15	6.4 (6.6) 1, 2, 4, 8, 19	6.6 (3.1) 3, 4, 6, 8, 12	5.2 (4.6) 1, 2, 4, 7, 14	6.5 (6.2) 1, 2, 5, 9, 19
0.25	5.8 (3.1) 2, 4, 5, 7, 12	5.0 (4.9) 1, 2, 4, 6, 14	6.1 (6.2) 1, 2, 4, 8, 18	6.4 (3.0) 3, 4, 6, 8, 12	5.0 (4.4) 1, 2, 4, 7, 14	6.3 (6.0) 1, 2, 4, 8, 18
0.5	5.3 (2.8) 2, 3, 5, 7, 10	4.5 (4.2) 1, 2, 3, 6, 12	5.3 (5.3) 1, 2, 4, 7, 16	5.8 (2.7) 2, 4, 5, 7, 11	4.5 (3.8) 1, 2, 3, 6, 12	5.4 (5.1) 1, 2, 4, 7, 15
0.75	4.7 (2.4) 2, 3, 4, 6, 9	3.7 (3.3) 1, 2, 3, 5, 10	4.4 (4.3) 1, 2, 3, 6, 13	5.2 (2.4) 2, 3, 5, 6, 10	3.7 (3.0) 1, 2, 3, 5, 10	4.4 (4.0) 1, 2, 3, 6, 12
1	4.1 (2.1) 2, 3, 4, 5, 8	3.1 (2.5) 1, 1, 2, 4, 8	3.5 (3.2) 1, 1, 2, 5, 10	4.5 (2.0) 2, 3, 4, 6, 8	3.1 (2.3) 1, 1, 2, 4, 8	3.5 (3.0) 1, 1, 3, 5, 9
1.5	3.1 (1.4) 1, 2, 3, 4, 6	2.1 (1.5) 1, 1, 2, 3, 5	2.3 (1.8) 1, 1, 2, 3, 6	3.3 (1.4) 1, 2, 3, 4, 6	2.1 (1.4) 1, 1, 2, 3, 5	2.3 (1.7) 1, 1, 2, 3, 6
2	2.3 (1.0) 1, 2, 2, 3, 4	1.6 (0.9) 1, 1, 1, 2, 3	1.7 (1.1) 1, 1, 1, 2, 4	2.5 (1.0) 1, 2, 2, 3, 4	1.6 (0.9) 1, 1, 1, 2, 3	1.7 (1.0) 1, 1, 1, 2, 4
3	1.6 (0.6) 1, 1, 2, 2, 3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	1.7 (0.6) 1, 1, 2, 2, 3	1.1 (0.4) 1, 1, 1, 1, 2	1.2 (0.4) 1, 1, 1, 1, 2

5.4 Effect of sample size

When checking the robustness of the LP-FK and LP-SR CUSUM charts, a conclusion was that they could work for some specific scenarios. One of them involved increasing the reference sample size “m” to values higher than 100. Since there was not enough time for this study, it should go as a future work recommendation.

For the LP-M Chart though, it doesn’t make much of a difference. In fact, comparing the cases where the CL chart outperforms the LP-M chart (in Tables 5.7 to 5.9), the one proposed by Chowdhury et al. (2015) is slightly better for m=100:

- For the Normal distribution, the CL Chart has a better performance 15 times when m=50 and 20 times when m=100.

- For the Lognormal distribution, the CL Chart has a better performance 31 times when m=50 and 40 times when m=100.

- For the Cauchy distribution, the CL Chart has a better performance 43 times when m=50 and 66 times when m=100.

Though we cannot recommend the CL Chart over the LP-M Chart for m=100 because the latter still has a better overall performance, it’s been shown that the reference sample size was an issue for the non-robust CUSUM charts, not for the distribution-free CUSUM chart LP-M.

5.5 Effect of k

Choosing the values of “k” equal to 0, 3 and 6 have a practical application. Referring to the work of Lepage (1971) and Chowdhury et al. (2015), the Lepage statistic follows a Chi-square distribution with 2 degrees of freedom for an In-control process. If the IC variance of the Lepage statistic is 4, the standard deviation is 2. Therefore, the values of 0, 3 and 6 correspond to 0, 1.5 and 3 times the SD. A change of 3 times the standard deviation is significant enough to cover a broad range of shifts related to real data.

Looking at Table 4.3, as “k” increases, the value of the Control limits H_t decrease deeply. There is also a direct relationship with the SDRL; as “k” goes up, the SDRL tends to go down, but in this case the decrease is more subtle.

Regarding the relationship between the performance of the proposed LP-M Chart and the value of k, there are some points to remark:

- For the Normal distribution, when k=0, the LP-M Chart outperforms the CL Chart in more cases than if k=3 or k=6. The LP-M Chart wins 96% of the cases when k=0, 82% when k=3 and 84% when k=6.

- For the Lognormal distribution the same conclusion can be extracted, k=0 is the best case for the chart proposed in this study. The LP-M Chart wins 95% of the cases when k=0, 80% when k=3 and 85% when k=6.

- For the Cauchy distribution, the one with the worst performance given by the LP-M Chart, the trend changes. The LP-M Chart wins 96% of the cases when k=0, 48% when k=3 and 43% when k=6.

- As “k” goes up, so does the ARL. Ergo, the Control Chart detects changes slower. This case is especially notorious for downward shifts in the scale parameter (δ=0.5). With k=0, the ARL stayed under 200 for any distribution used. An upper limit of 3000 had to be introduced in the “GetARL” algorithm for the cases of k=3 and k=6, since simulations never ended.

This analysis shows the weak point of the proposed Control Chart. For a symmetric heavy-tailed large distribution (represented by the Cauchy in this study) and a large value of standard deviation in the Lepage statistic, the performance shown by the LP-M Chart is actually worse than the CL Chart. Therefore, the Ansari-Bradley statistic should be recommended for monitoring cases where k=3 and k=6 and data follows a heavy-tailed distribution.

Chapter 6. Conclusions and future research

6.1 Study conclusions

In this study, three versions of the CUSUM-Lepage Control Chart by Chowdhury et al. (2015) were proposed; the LP-FK Chart, LP-SR Chart and LP-M Chart based on the statistics submitted by Conover et al. (1981) and analyzed by Guerrero (2016). The first two charts do not confirm the hypotheses one and two made in Chapter 1, as they do not show robustness (considering robust as showing a maximal deviation of 10% from the expected ARLo). Therefore, they cannot be recommended for general usage for monitoring changes in both location and scale of processes, though they could be useful for certain specific scenarios.

On the other hand, the LP-M Chart confirms the third hypotheses since it has shown better overall performance than the CL Chart, consisting of the WRS and Mood statistics combined into a Lepage-type statistic. The Mood statistic is a substitution of the A-B statistic used by Chowdhury to detect changes in the scale parameter.

The following sections cover a table to help deciding which Control Chart should perform better for a variety of sample sizes and distributions, and afterwards some recommendations on how this work can be expanded will be made.

6.2 Recommendations for practitioners: decision table

From all analyzed scenarios in the previous Chapter, Tables 6.1 to 6.3 highlight which Control Chart should be used in first place. The best performing chart is shown first in the “Proposed chart” column, the second best goes right behind it and so on. The analysis is divided in three tables since Table 6.3 is a summary for $\delta \geq 1.25$, because the behavior is the same in the $\delta \in [1.25, 2]$ range.

As seen in the three tables, the LP-FK and LP-SR CUSUM charts can only be recommended for certain cases, since for some scenarios they lack the necessary robustness. The LP-FK Chart can be used when data follows a Normal distribution or when k=0 or m=100 for a Cauchy distribution. The LP-SR can be employed also under a Normal distribution, but only for the triplets (50, 5, 0) & (50, 5, 3) under a Cauchy distribution. None of them can be used when data follows a Lognormal distribution, though.

The CL chart proposed by Chowdhury et al. (2015) can still be a better option than the proposed charts for downward shifts in scale (δ=0.5), or for δ=1 with data under a Cauchy distribution. However, for the rest of cases the LP-M Chart is the best performing one.

Table 6.1 Control charts decision table for δ=0.5

Distribution	m	k	Proposed chart
Normal	50/100	0/3/6	CL, LP-M, LP-FK, LP-SR
Cauchy	50	0	CL, LP-M, LP-FK, LP-SR
	50	3	CL, LP-M, LP-SR
	50	6	CL, LP-M
	100	0	CL, LP-M, LP-FK
	100	3	CL, LP-M, LP-FK
	100	6	CL, LP-M, LP-FK
Lognormal	50/100	0/3/6	LP-M, CL

Table 6.2 Control charts decision table for δ=1

Distribution	m	k	Proposed chart
Normal	50/100	0/3/6	LP-M, LP-FK, LP-SR, CL
Cauchy	50	0	LP-M, CL, LP-FK, LP-SR
	50	3	CL, LP-M, LP-SR
	50	6	CL, LP-M
	100	0	CL, LP-M, LP-FK
	100	3	CL, LP-M, LP-FK
	100	6	CL, LP-M, LP-FK
Lognormal	50/100	0/3/6	LP-M, CL

Table 6.3 Control charts decision table for δ ≥1.25

Distribution	m	k	Proposed chart
Normal	50/100	0/3/6	LP-M, LP-FK, LP-SR, CL
Cauchy	50	0	LP-M, LP-FK, LP-SR, CL
	50	3	LP-M, LP-SR, CL
	50	6	LP-M, CL
	100	0	LP-M, LP-FK, CL
	100	3	LP-M, LP-FK, CL
	100	6	LP-M, LP-FK, CL
Lognormal	50/100	0/3/6	LP-M, CL

6.3 Future work

In this work, all the material presented by Chowdhury in his paper was reproduced using three different statistics. There are still some tests recommended by Guerrero (2016) like the Talwar and Gentle (T-G), or the nonparametric version of the Levene test, the N-P-L statistic, which could be combined with the WRS statistic into CUSUM Charts. The T-G statistic showed an overall good performance, and the N-P-L test is actually distribution-free and was useful for skewed distributions. However, both F-K and S-R also showed good performance but they cannot be recommended for general use in this work. When the latter two have been implemented into a Control Chart, they haven't shown the expected robustness to work under a variety of scenarios.

However, the LP-FK and LP-SR statistic showed some good results for some cases; it would be an interesting choice to analyze the robustness of all proposed charts with bigger reference and test samples. The values m=150 and n=11 would be a good start for it.

There hasn't been an accurate analysis on how does the reference sample size affect the performance of the chart. Therefore, more work could be done to determine with accuracy the best range of sample sizes for each test.

Last but not least, this study only covered the Cumulative sum Control Charts. The Lepage-type statistic technique can be implemented in other charts as the Exponentially Weighted Moving Chart (EWMA Chart), the Cuscore Chart or a Change-point methodology, to name a few of them.

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Appendix A: Glossary of abbreviations and acronyms

Table A.1 Abbreviations

	Description
A-B	**A**nsari **B**radley
ARL	**A**verage **R**un **L**ength
CL	**CUSUM L**epage
EWMA	**E**xponentially **W**eighted **M**oving **A**verage
F-K	**F**ligner and **K**illeen
IC	**I**n **c**ontrol
IID	**I**ndependent and **I**dentically **D**itributed
LP	**L**e**p**age
LP-FK	**L**e**p**age – **F**ligner and **K**illeen
LP-M	**L**e**p**age – **M**ood
LP-SR	**L**e**p**age – **S**quared **R**anks
MW	**M**ann **W**hitney
NPL	**N**on-**P**arametric **L**evene
OOC	**O**ut of **c**ontrol
RL	**R**un **L**ength
RL P%	**R**un **L**ength **P**ercentiles
SD	**S**tandard **D**eviation
SDRL	**S**tandard **D**eviation **R**un **L**ength
SPC	**S**tatistical **P**rocess **C**ontrol
S-R	**S**quared-**R**anks
T-G	**T**alwar and **G**entle
UCL	**U**pper **C**ontrol **L**imit
WRS	**W**ilcoxon **R**ank **S**um

Table A.2 Acronyms

	Description
Cuscore	Cumulative Score
CUSUM	Cumulative Sum

Appendix B: OOC performance characteristics

LP-FK chart Normal distribution OOC performance

	m=50, n=5			m=100, n=5		
	MW-FK CUSUM chart			MW-FK CUSUM chart		
θ	K=0	K=3	K=6	K=0	K=3	K=6
δ = 0.5						
0	103.24894 (403.780126475644) 9, 14, 21, 38.25, 251	2796.11604 (664.315144992913) 740.95, 3001, 3001, 3001, 3001	2975.52048 (234.826790322141) 3001, 3001, 3001, 3001, 3001	57.66478 (272.429182505732) 13, 19, 27, 41, 104.056	2983.72342 (193.700347753942) 3001, 3001, 3001, 3001, 3001	2999.08562 (63.0858396605682) 3001, 3001, 3001, 3001, 3001
0.25	56.22002 (261.268709412302) 8, 13, 19, 30, 96	2652.65896 (838.45036191292) 296, 3001, 3001, 3001, 3001	2802.12252 (652.218999647603) 796, 3001, 3001, 3001, 3001	35.34938 (113.786634107779) 13, 19, 25, 35, 70	2939.42258 (359.645221322136) 3001, 3001, 3001, 3001, 3001	2948.94466 (330.918561128515) 3001, 3001, 3001, 3001, 3001
0.5	17.20784 (51.330381049577) 6, 10, 13, 18, 34	1783.95774 (1303.60801078814) 14, 265, 2630.5, 3001, 3001	1944.5859 (1265.06499687965) 25, 444, 3001, 3001, 3001	19.61552 (9.64478488962358) 10, 14, 18, 23, 35	2134.53978 (1183.66352957442) 54, 858, 3001, 3001, 3001	2257.99638 (1111.22246816485) 95, 1277, 3001, 3001, 3001
0.75	8.67034 (4.59190221974727) 3, 6, 8, 10, 16	524.81562 (971.960723248098) 2, 9, 42, 377, 3001	702.7302 (1078.47424654554) 2, 19, 109, 825, 3001	10.50038 (4.14044233396076) 5, 8, 10, 13, 18	432.244 (852.27980703604) 3, 12, 48, 295, 3001	673.72166 (999.464586934452) 5, 36, 161, 792, 3001
1	4.95588 (2.31725717589058) 2, 3, 5, 6, 9	53.31638 (290.477590165409) 1, 2, 4, 11, 129	113.6446 (422.150290332948) 1, 3, 8, 35, 472.049999999996	5.618 (2.21538578100591) 3, 4, 5, 7, 10	16.13614 (103.319355126149) 1, 3, 4, 9, 37	57.21756 (229.834502600498) 1, 3, 9, 29, 201
1.5	2.11872 (0.81968225501104) 1, 2, 2, 2, 4	1.44398 (1.49150469369862) 1, 1, 1, 2, 3	1.859 (5.632032794282) 1, 1, 1, 2, 4	2.296 (0.730503027162416) 1, 2, 2, 3, 4	1.38472 (0.633970472475808) 1, 1, 1, 2, 3	1.49228 (1.29871249115526) 1, 1, 1, 2, 3
2	1.26 (0.449715515499407) 1, 1, 1, 2, 2	1.0165 (0.130950727229211) 1, 1, 1, 1, 1	1.01732 (0.154597851246103) 1, 1, 1, 1, 1	1.35188 (0.483926801596053) 1, 1, 1, 2, 2	1.01116 (0.105430909668533) 1, 1, 1, 1, 1	1.00938 (0.0982456545816296) 1, 1, 1, 1, 1
3	1.00016 (0.0126482251542258) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1
δ = 1						
0	476.0875 (2153.71454857204) 13, 39, 101, 303, 1738	***496.5913 (1466.23762087853) 8, 49, 150, 437, 1926.01	***499.45162 (1537.27765748133) 9, 53, 158, 445, 1908	507.7668 (2517.35787717031) 21, 61, 144, 384, 1815	509.49008 (1183.95472213986) 13, 77, 216, 549, 1878	239.48012 (364.410521342821) 8, 43, 116, 279, 881
0.25	219.77018 (1144.21119004654) 8, 22, 47, 127, 727.056	286.36074 (940.31730241405) 4, 25, 79, 232, 1089	287.4708 (883.820885074854) 5, 28, 86, 249, 1114	163.79458 (620.516162036712) 12, 29, 57, 128, 551	***248.70206 (527.791741593411) 6, 36, 101, 260, 934.056	56.19388 (86.8058645237055) 2, 11, 30, 68, 194
0.5	35.9274 (152.449007682041) 4, 9, 16, 31, 101	78.80156 (268.00492494423) 2, 8, 23, 65, 298.056	84.17794 (321.583925596073) 2, 9, 27, 72, 302.056	26.89224 (37.833789338298) 6, 12, 19, 31, 70	***58.55824 (115.000895440955) 2, 10, 25, 63, 214	16.4191 (21.8732572786423) 1, 4, 10, 20, 54
0.75	10.34358 (14.1154425257572) 2, 5, 8, 12, 25	18.79316 (47.7549581502796) 1, 3, 8, 18, 66	22.07666 (52.4007654528901) 1, 4, 9, 22, 79	10.35338 (6.78116378354374) 3, 6, 9, 13, 23	14.70852 (22.5014125263485) 1, 4, 8, 17, 49	6.1837 (6.80075873093483) 1, 2, 4, 8, 19
1	5.33052 (3.58292803124282) 2, 3, 5, 7, 12	6.21338 (11.1009492205429) 1, 2, 3, 7, 19	7.55686 (13.1805736404312) 1, 2, 4, 8, 25	5.65868 (3.002945389317) 2, 4, 5, 7, 11	5.19892 (5.64411445260982) 1, 2, 4, 6, 15	3.05216 (2.80308338797289) 1, 1, 2, 4, 8
1.5	2.45354 (1.18129986785217) 1, 2, 2, 3, 5	1.8091 (1.28452722439184) 1, 1, 1, 2, 4	1.98198 (1.75802078837022) 1, 1, 1, 2, 5	2.61002 (1.12793663138533) 1, 2, 2, 3, 5	1.76534 (1.07830326863957) 1, 1, 1, 2, 4	1.36972 (0.724401557668352) 1, 1, 1, 2, 3
2	1.54528 (0.637838426529138) 1, 1, 1, 2, 3	1.15858 (0.42411788587564) 1, 1, 1, 1, 2	1.16912 (0.472189458647371) 1, 1, 1, 1, 2	1.63564 (0.642160445528029) 1, 1, 2, 2, 3	1.15146 (0.392024160044813) 1, 1, 1, 1, 2	1.0539 (0.236465448465873) 1, 1, 1, 1, 2
3	1.03042 (0.171858122602006) 1, 1, 1, 1, 1	1.00138 (0.0371230812630532) 1, 1, 1, 1, 1	1.00108 (0.0328459309063321) 1, 1, 1, 1, 1	1.03658 (0.187730147957564) 1, 1, 1, 1, 1	1.0009 (0.0299867968306) 1, 1, 1, 1, 1	1.00012 (0.0109539034028969) 1, 1, 1, 1, 1
δ = 1.25						
0	35.1767 (72.7551294637517) 5, 11, 19, 36, 109	50.98458 (92.102431017751) 2, 9, 24, 57, 184.056	56.8939 (102.467393419196) 2, 10, 27, 64, 204	29.57636 (30.8050377074491) 6, 13, 22, 35, 77	48.78428 (70.6066615192548) 3, 11, 27, 59, 165	54.83128 (74.1504528600789) 3, 12, 31, 69, 184
0.25	25.87006 (47.6183986063592) 4, 9, 16, 28, 74	38.8746 (74.3421770600364) 2, 7, 18, 43, 140	42.7652 (77.3116879320444) 2, 8, 20, 49, 153	22.68036 (21.262025109209) 5, 11, 17, 28, 57	34.95488 (49.5959663620257) 2, 8, 19, 42, 119	40.72686 (56.6878244840045) 2, 9, 23, 50, 137
0.5	13.77664 (16.4837466793351) 3, 6, 10, 16, 36	19.27594 (33.8078360223178) 1, 4, 9, 21, 68	22.0047 (38.0325224954295) 1, 4, 11, 25, 77	13.25052 (9.6478775572883) 3, 7, 11, 17, 31	16.72564 (22.3803910651375) 1, 4, 10, 21, 54	20.01414 (26.0720113350677) 1, 5, 12, 25, 66
0.75	7.7648 (6.40771894961403) 2, 4, 6, 10, 18	8.78102 (12.6700733373871) 1, 2, 5, 10, 29	10.15856 (14.6837968865079) 1, 2, 6, 12, 34	7.88986 (4.80126548145429) 2, 5, 7, 10, 17	7.7132 (8.74630183082623) 1, 3, 5, 10, 23	9.34056 (10.9989744265167) 1, 3, 6, 12, 29
1	4.8796 (3.13148845828933) 1, 3, 4, 6, 11	4.55732 (5.52884126913728) 1, 2, 3, 5, 13	5.27898 (6.73728862234353) 1, 2, 3, 6, 17	5.12164 (2.78514610948457) 2, 3, 5, 6, 10	4.12418 (3.89946450734827) 1, 2, 3, 5, 11	4.87424 (5.13212540733602) 1, 2, 3, 6, 14
1.5	2.56348 (1.29616507069038) 1, 2, 2, 3, 5	1.90672 (1.37970174789273) 1, 1, 1, 2, 4	2.04388 (1.68196644610483) 1, 1, 1, 2, 5	2.69534 (1.24149631923023) 1, 2, 2, 3, 5	1.84888 (1.17159301730762) 1, 1, 2, 2, 4	1.978 (1.43956849338576) 1, 1, 1, 2, 5
2	1.68348 (0.731126377937228) 1, 1, 2, 2, 3	1.25424 (0.553469194231241) 1, 1, 1, 1, 2	1.2751 (0.625705857537232) 1, 1, 1, 1, 2	1.77274 (0.734046094515445) 1, 1, 2, 2, 3	1.24464 (0.514233953693436) 1, 1, 1, 1, 2	1.25044 (0.558682422240917) 1, 1, 1, 1, 2
3	1.0882 (0.284925224660213) 1, 1, 1, 1, 2	1.00942 (0.0974240905991356) 1, 1, 1, 1, 1	1.00902 (0.0951778376325893) 1, 1, 1, 1, 1	1.10572 (0.308715383004725) 1, 1, 1, 1, 2	1.0075 (0.0869706920636232) 1, 1, 1, 1, 1	1.00778 (0.088088743846963) 1, 1, 1, 1, 1

δ = 1.5						
0	11.3033 (9.67114779806382) 3, 6, 9, 14, 28	14.32184 (19.2275014744677) 1, 4, 8, 17, 47	16.61426 (22.2545206626923) 1, 4, 10, 21, 55	11.12818 (7.08483406898677) 3, 6, 10, 14, 24	12.52344 (14.21113963397) 1, 4, 8, 16, 38	15.27878 (17.6138680214919) 1, 4, 10, 20, 48
0.25	10.23024 (8.56640515084035) 3, 5, 8, 13, 25	12.287 (16.3639893242122) 1, 3, 7, 15, 41	14.27698 (19.2929569129081) 1, 4, 8, 18, 47	10.10444 (6.23704018744516) 3, 6, 9, 13, 22	10.86366 (12.3078601337853) 1, 3, 7, 14, 33	13.39406 (15.3244586689733) 1, 4, 8, 17, 42
0.5	7.8286 (5.66223836118413) 2, 4, 6, 10, 18	8.5297 (10.7605350095743) 1, 2, 5, 10, 27	9.90458 (12.9690168837726) 1, 3, 6, 12, 32	7.98452 (4.74796495701313) 2, 5, 7, 10, 17	7.61388 (7.93112913788371) 1, 3, 5, 10, 23	9.15274 (10.2012181522324) 1, 3, 6, 12, 28
0.75	5.76576 (3.86153206007629) 2, 3, 5, 7, 13	5.53116 (6.26170050643311) 1, 2, 4, 7, 17	6.40466 (7.80559086997112) 1, 2, 4, 8, 20	5.96872 (3.35269538922454) 2, 4, 5, 8, 12	5.01228 (4.76431980207705) 1, 2, 4, 6, 14	5.85798 (5.95974837485727) 1, 2, 4, 8, 17
1	4.27546 (2.57402297975096) 1, 3, 4, 5, 9	3.58662 (3.44366289753847) 1, 1, 3, 4, 10	4.06588 (4.2401768132459) 1, 1, 3, 5, 12	4.46138 (2.35389449890734) 2, 3, 4, 6, 9	3.36982 (2.81185549028146) 1, 1, 3, 4, 9	3.83108 (3.59090015523664) 1, 1, 3, 5, 11
1.5	2.58618 (1.32069977381608) 1, 2, 2, 3, 5	1.93656 (1.34292644453399) 1, 1, 2, 2, 4	2.05592 (1.63386852106492) 1, 1, 1, 2, 5	2.71576 (1.27128279528943) 1, 2, 2, 3, 5	1.86864 (1.17463702727392) 1, 1, 2, 2, 4	1.9629 (1.39095732689601) 1, 1, 1, 2, 5
2	1.7943 (0.808035004446817) 1, 1, 2, 2, 3	1.3304 (0.639502947045896) 1, 1, 1, 2, 3	1.36412 (0.737446609694589) 1, 1, 1, 2, 3	1.87584 (0.803714634397229) 1, 1, 2, 2, 3	1.32008 (0.603121935201642) 1, 1, 1, 2, 2	1.33494 (0.667415991179493) 1, 1, 1, 2, 3
3	1.16716 (0.381812061109913) 1, 1, 1, 1, 2	1.0292 (0.171778782621729) 1, 1, 1, 1, 1	1.02874 (0.172147044970577) 1, 1, 1, 1, 1	1.1973 (0.407941219198019) 1, 1, 1, 1, 2	1.02532 (0.159120721428982) 1, 1, 1, 1, 1	1.0245 (0.157544426765684) 1, 1, 1, 1, 1
δ = 1.75						
0	6.56372 (4.18834700169555) 2, 4, 6, 8, 14	6.5651 (7.24399140655412) 1, 2, 4, 8, 20	7.63722 (8.6028629453582) 1, 2, 5, 10, 24	6.667 (3.62360781689633) 2, 4, 6, 8, 13	5.82532 (5.59882075382635) 1, 2, 4, 7, 16	7.03208 (7.13678005449173) 1, 2, 5, 9, 21
0.25	6.2428 (3.91524133895918) 2, 4, 5, 8, 14	6.05878 (6.52668498274103) 1, 2, 4, 8, 18	7.13264 (8.06026959780625) 1, 2, 5, 9, 22	6.344 (3.45640607204343) 2, 4, 6, 8, 13	5.4035 (5.03463550826612) 1, 2, 4, 7, 15	6.5094 (6.56133009846732) 1, 2, 4, 9, 19
0.5	5.45874 (3.34464965968439) 2, 3, 5, 7, 12	5.01576 (5.16163195813973) 1, 2, 3, 6, 14	5.84564 (6.36331225269172) 1, 2, 4, 7, 18	5.60258 (3.01260333585044) 2, 3, 5, 7, 11	4.51666 (4.00685956116144) 1, 2, 3, 6, 12	5.35454 (5.26615571705972) 1, 2, 4, 7, 15
0.75	4.55222 (2.64071439937203) 1, 3, 4, 6, 10	3.9072 (3.6689313677347) 1, 2, 3, 5, 11	4.3736 (4.46277731548722) 1, 1, 3, 6, 13	4.70574 (2.45891423155467) 2, 3, 4, 6, 9	3.56902 (2.97718886076618) 1, 2, 3, 5, 9	4.06604 (3.74738303032817) 1, 2, 3, 5, 11
1	3.73398 (2.07478660454252) 1, 2, 3, 5, 8	2.96148 (2.54645751948034) 1, 1, 2, 4, 8	3.31642 (3.13169196018553) 1, 1, 2, 4, 9	3.87262 (1.97849251306455) 1, 2, 4, 5, 8	2.79994 (2.13269945480822) 1, 1, 2, 4, 7	3.10156 (2.66828558467028) 1, 1, 2, 4, 8
1.5	2.54778 (1.27890178801801) 1, 2, 2, 3, 5	1.8952 (1.27757958820266) 1, 1, 1, 2, 4	2.004 (1.50368521352856) 1, 1, 1, 2, 5	2.66356 (1.26201425506989) 1, 2, 2, 3, 5	1.84458 (1.15442248653842) 1, 1, 1, 2, 4	1.91764 (1.33013240523051) 1, 1, 1, 2, 5
2	1.86688 (0.856675985116234) 1, 1, 2, 2, 3	1.39026 (0.705611146554391) 1, 1, 1, 2, 3	1.41706 (0.788373887865232) 1, 1, 1, 2, 3	1.95282 (0.862466767231348) 1, 1, 2, 2, 3	1.37252 (0.660482834259519) 1, 1, 1, 2, 3	1.3904 (0.723904911412) 1, 1, 1, 2, 3
3	1.24098 (0.450946456526862) 1, 1, 1, 1, 2	1.05696 (0.239909794577443) 1, 1, 1, 1, 2	1.05574 (0.245263644850006) 1, 1, 1, 1, 2	1.2808 (0.474421607346493) 1, 1, 1, 2, 2	1.0519 (0.227612447261838) 1, 1, 1, 1, 2	1.04932 (0.224963440965405) 1, 1, 1, 1, 1
δ = 2						
0	4.72556 (2.65615959424049) 2, 3, 4, 6, 10	4.039 (3.73409130428775) 1, 2, 3, 5, 11	4.66382 (4.66678032313197) 1, 2, 3, 6, 13	4.80186 (2.41293949073318) 2, 3, 4, 6, 9	3.65814 (2.97848437427215) 1, 2, 3, 5, 9	4.26216 (3.90923749306122) 1, 2, 3, 6, 12
0.25	4.57744 (2.54161606908771) 2, 3, 4, 6, 9	3.86372 (3.50266086486584) 1, 2, 3, 5, 10	4.47104 (4.48035297601158) 1, 2, 3, 6, 13	4.67162 (2.35284026077969) 2, 3, 4, 6, 9	3.5409 (2.90346961284527) 1, 2, 3, 5, 9	4.11632 (3.75228880165493) 1, 2, 3, 5, 11
0.5	4.2475 (2.34441329024417) 1, 3, 4, 5, 9	3.48306 (3.08398820621369) 1, 1, 3, 4, 9	3.99206 (3.88073422928744) 1, 1, 3, 5, 11	4.31054 (2.14449921565332) 2, 3, 4, 5, 8	3.20026 (2.54294421167773) 1, 1, 2, 4, 8	3.6239 (3.21604969636867) 1, 1, 3, 5, 10
0.75	3.78048 (2.04363756539411) 1, 2, 3, 5, 8	2.99318 (2.51316131738417) 1, 1, 2, 4, 8	3.36968 (3.11310303809728) 1, 1, 2, 4, 9	3.88252 (1.93642801174454) 1, 3, 4, 5, 7	2.7962 (2.08489627939855) 1, 1, 2, 4, 7	3.11086 (2.64117958060249) 1, 1, 2, 4, 8
1	3.29356 (1.73644545876242) 1, 2, 3, 4, 7	2.53978 (1.97777040707855) 1, 1, 2, 3, 6	2.7602 (2.40517185602882) 1, 1, 2, 3, 7	3.39482 (1.65846681541825) 1, 2, 3, 4, 6	2.38622 (1.68979028859335) 1, 1, 2, 3, 6	2.59642 (2.07220391472009) 1, 1, 2, 3, 7
1.5	2.47436 (1.21898823170947) 1, 2, 2, 3, 5	1.81892 (1.18025331753733) 1, 1, 1, 2, 4	1.9315 (1.39883054175182) 1, 1, 1, 2, 5	2.55002 (1.19656451348546) 1, 2, 2, 3, 5	1.768 (1.06963492948653) 1, 1, 1, 2, 4	1.83354 (1.22913843159576) 1, 1, 1, 2, 4
2	1.92122 (0.88303414802947) 1, 1, 2, 2, 4	1.42312 (0.731997392074344) 1, 1, 1, 2, 3	1.45502 (0.831811660060591) 1, 1, 1, 2, 3	1.98554 (0.879742227773424) 1, 1, 2, 2, 4	1.39478 (0.683270143326763) 1, 1, 1, 2, 3	1.4236 (0.766664721697764) 1, 1, 1, 2, 3
3	1.3124 (0.504986475290977) 1, 1, 1, 2, 2	1.0859 (0.296383108255071) 1, 1, 1, 1, 2	1.08674 (0.307470426482391) 1, 1, 1, 1, 2	1.35582 (0.528940188636166) 1, 1, 1, 2, 2	1.0788 (0.281553095246543) 1, 1, 1, 1, 2	1.07546 (0.27972728388033) 1, 1, 1, 1, 2

LP-FK chart Cauchy distribution OOC performance

θ	m=50, n=5 MW-FK CUSUM chart K=0	m=50, n=5 MW-FK CUSUM chart K=3	m=50, n=5 MW-FK CUSUM chart K=6	m=100, n=5 MW-FK CUSUM chart K=0	m=100, n=5 MW-FK CUSUM chart K=3	m=100, n=5 MW-FK CUSUM chart K=6
δ = 0.5						
0	182.65078 (441.657933767753) 14, 27, 51, 121, 790.049999999996	1741.4749 (1152.10979219407) 88, 580, 1746, 3001, 3001	1796.17002 (1155.17405657122) 91, 616, 1906, 3001, 3001	171.59244 (361.9389459161) 23, 42, 71, 141, 604	2155.55574 (1062.83166986886) 186, 1168, 3001, 3001, 3001	2139.5322 (1068.05256798012) 182, 1126, 3001, 3001, 3001
0.25	152.17768 (392.221027402285) 13, 25, 45, 98, 596	1635.48168 (1165.67747420282) 66, 472, 1486, 3001, 3001	1696.9166 (1171.20458402791) 73, 508, 1645, 3001, 3001	146.5087 (314.163514015522) 22, 40, 66, 123, 472.049999999996	2057.70356 (1094.76423879212) 155, 986, 2807, 3001, 3001	2042.29224 (1098.79800978028) 153, 963, 2750, 3001, 3001
0.5	95.84696 (287.938473966322) 11, 20, 34, 65, 289	1361.1798 (1172.38133190174) 31, 261.75, 944, 3001, 3001	1399.40846 (1174.2859893621) 37, 286, 1008, 3001, 3001	99.21554 (228.036460538781) 18, 32, 51, 88, 273	***1716.83058 (1159.80303656204) 77, 544, 1700, 3001, 3001	1724.4127 (1151.04504344207) 85, 570, 1704.5, 3001, 3001
0.75	55.80246 (201.873745341314) 7, 14, 23, 40, 136	987.53734 (1104.79535854447) 11, 101, 444, 1698.25, 3001	1039.25296 (1108.84600021837) 16, 133, 509, 1847, 3001	***53.9089 (131.196310657841) 12, 21, 32, 52, 134	1224.13146 (1133.84109597901) 28, 219, 763, 2389, 3001	1275.65688 (1119.62472767409) 41, 274, 857, 2462, 3001
1	31.7568 (139.192487242416) 5, 9, 15, 25, 68	643.50492 (957.865048284365) 4, 30, 159, 775, 3001	718.44734 (978.169505749675) 7, 57, 234, 927, 3001	27.87046 (59.1425031267863) 8, 13, 19, 30, 65	723.89896 (963.342444307432) 8, 60, 253, 965, 3001	850.47468 (983.395750195682) 17, 123, 399, 1229, 3001
1.5	11.96244 (63.6232459687098) 3, 5, 7, 11, 26	209.4261 (572.115035575074) 2, 5, 15, 85.25, 1332	281.46832 (622.180399149121) 2, 10, 43, 199, 1709	10.54322 (20.2783834732536) 4, 6, 9, 12, 22	144.16368 (430.231315071959) 2, 6, 17, 66, 724	276.5225 (544.429616319587) 3, 19, 71, 249, 1357.05
2	5.99576 (24.8232484960313) 2, 3, 5, 6, 13	59.65754 (297.408323446681) 1, 2, 4, 11, 179	95.70424 (335.677569454134) 1, 3, 9, 41, 419	5.91862 (2.82463393499712) 3, 4, 5, 7, 11	20.09288 (128.111208810713) 1, 3, 4, 9, 45	72.86496 (236.230493588614) 1, 4, 12, 45, 304
3	3.02182 (6.64392254031716) 2, 2, 3, 3, 5	5.77194 (72.5451855378101) 1, 1, 2, 3, 7	13.90696 (105.981970103967) 1, 1, 2, 4, 34	3.31218 (1.09722728992425) 2, 3, 3, 4, 5	2.32936 (11.7957884349837) 1, 1, 2, 3, 5	5.71928 (36.1411759627935) 1, 1, 2, 4, 13
δ = 1						
0	457.27584 (2779.78669939155) 13, 39, 97, 282, 1611.05	396.96242 (1042.70229711522) 8, 48, 144, 386, 1509	401.5968 (1113.29086729429) 9, 51, 149, 396, 1519	480.47082 (2221.06596997689) 22, 61, 142, 375, 1744	464.59408 (824.876205187197) 13, 78, 213, 519, 1723	467.14496 (793.178043048821) 14, 81, 219, 534, 1697.05
0.25	373.10684 (1930.37233984862) 11, 32, 77, 232, 1314	368.02306 (945.741274143932) 7, 43, 126, 349, 1408	387.91602 (1052.41041754683) 8, 47, 137, 369, 1451	362.0599 (1322.27516294609) 18, 48, 107, 279, 1330	424.98456 (813.464138557872) 12, 68, 191, 474, 1561.05	424.69538 (724.946888390091) 12, 74, 201, 484, 1524
0.5	243.35712 (1622.75185574906) 8, 21, 45, 125, 805	297.50968 (764.962670268318) 5, 31, 95, 279, 1161.05	316.84214 (1000.79008727914) 6, 36, 105, 294, 1208	186.30876 (751.297801404337) 13, 30, 57, 131, 625	327.99102 (638.140218383175) 8, 49, 138, 350, 1246.05	345.09774 (639.012897566675) 9, 56, 154, 380, 1264
0.75	120.15656 (801.340549543384) 6, 13, 26, 60, 360	222.85626 (682.189079506273) 4, 19, 60, 183, 884.056	235.79436 (683.603849109) 4, 23, 70, 202, 919.056	83.05536 (988.716166393786) 9, 18, 31, 59, 217	218.8905 (464.533471621449) 6, 30, 85, 224, 830	245.13368 (464.888744067379) 7, 37, 104, 265, 913
1	62.82412 (664.988250553115) 4, 9, 16, 31, 138	151.82258 (551.51431588215) 2, 11, 34, 110, 598	164.0351 (581.193555630972) 3, 14, 45, 133.25, 626	33.65924 (158.016720344084) 6, 12, 19, 32, 81	130.87272 (300.072162097471) 3, 16, 45, 125, 511.056	163.0596 (344.976631477214) 4, 23, 65, 168, 619
1.5	20.0494 (1038.43095145342) 3, 5, 8, 13, 32	60.21714 (382.465788001751) 1, 4, 10, 32, 210	76.01358 (508.36957571052) 1, 5, 16, 50, 273	11.37922 (9.51364821620222) 4, 7, 9, 13, 25	36.90134 (115.059099501656) 2, 5, 12, 31, 135	62.37044 (144.747089395967) 2, 8, 23, 60, 238
2	49.18656 (9507.50395446167) 2, 3, 5, 7, 15	19.6669 (136.851603686689) 1, 2, 4, 10, 58	29.84404 (134.560642268857) 1, 3, 6, 19, 107	6.645 (3.5081478221453) 3, 4, 6, 8, 13	***10.42698 (38.6022676896192) 1, 3, 5, 10, 32	***22.86454 (58.723088462808) 1, 3, 8, 21, 86
3	3.47256 (33.2133736559566) 2, 2, 3, 4, 6	3.69378 (29.3417836560248) 1, 1, 2, 3, 8	6.46396 (85.4573430447495) 1, 1, 2, 4, 18	3.66372 (1.40373614725255) 2, 3, 3, 4, 6	***2.61712 (2.48203265798867) 1, 1, 2, 3, 6	***4.3253 (11.7446327587529) 1, 1, 2, 4, 13
δ = 1.25						
0	198.19472 (1569.38426336683) 8, 21, 46, 119, 651	171.83656 (382.34215854113) 4, 22, 65, 171, 659	175.4984 (379.856941470548) 4, 25, 70, 181, 662.056	144.92388 (351.922300333263) 12, 29, 56, 124, 515	194.66526 (336.394555239927) 6, 34, 91, 222, 709	206.59522 (338.816591545936) 7, 38, 101, 239, 748
0.25	169.6608 (947.762201824479) 7, 19, 40, 103, 567	167.57302 (963.276010311438) 4, 21, 59, 159, 622.056	169.59 (387.357953143701) 4, 24, 67, 172, 638	124.16096 (314.535302153835) 11, 25, 49, 105, 425.056	180.192 (317.547191296623) 6, 31, 83, 203, 654	193.20626 (324.125230767549) 6, 35, 93, 223, 700
0.5	127.9145 (784.244431753471) 6, 14, 29, 69, 393	136.63058 (361.750733821968) 3, 16, 47, 129, 526	148.07568 (409.369596650595) 3, 19, 55, 144, 562	78.33068 (208.879417997473) 9, 20, 35, 68, 244	147.30844 (288.227019068402) 5, 24, 66, 160, 541	161.19408 (274.796395453311) 5, 28, 77, 183, 585
0.75	77.37258 (538.408318689911) 5, 11, 20, 41, 204	105.4106 (299.412953043066) 2, 11, 33, 95, 406	116.4847 (327.291719484778) 3, 14, 41, 109, 441	43.68162 (125.634657859687) 7, 14, 24, 41, 120	104.63446 (208.16029413572) 3, 16, 44, 113, 393	124.12618 (220.920288009229) 4, 21, 57, 139, 456
1	42.95372 (369.589219623607) 4, 8, 14, 26, 100	76.40848 (232.41390566952) 2, 8, 22, 63, 296	87.48238 (262.673712957968) 2, 10, 28, 78, 338	24.55968 (52.1751270547113) 6, 10, 16, 26, 61	68.23308 (148.449433338086) 3, 10, 27, 69, 257	86.7453 (163.116648962432) 3, 14, 38, 94, 318
1.5	13.816 (150.050585628864) 3, 5, 8, 12, 30	33.37344 (149.869503235069) 1, 4, 9, 23, 118	42.43306 (140.931121583052) 1, 4, 12, 34, 158	11.02458 (11.2828117887536) 4, 6, 9, 13, 24	25.39152 (71.2787651782556) 2, 5, 10, 24, 88	39.45902 (91.9875930491993) 2, 6, 17, 41, 147
2	6.84618 (62.4996912283227) 2, 3, 5, 7, 15	14.02308 (73.5041539270162) 1, 2, 4, 10, 43	20.7623 (146.866223320616) 1, 2, 6, 15, 72	6.6792 (3.63720111413143) 3, 4, 6, 8, 13	***8.98324 (19.3581386980535) 1, 3, 5, 9, 28	***17.09882 (37.5537819711731) 1, 3, 7, 17, 61
3	3.43848 (1.84677110137397) 2, 2, 3, 4, 7	3.60274 (25.1290786521169) 1, 1, 2, 3, 8	5.30442 (28.1199061706482) 1, 1, 2, 4, 16	3.79386 (1.51992516389791) 2, 3, 4, 5, 7	***2.7506 (2.40993688629307) 1, 1, 2, 3, 7	***4.28378 (8.23631505282022) 1, 1, 2, 5, 12

δ = 1.5						
0	88.56246 (576.277973156007) 6, 13, 25, 55, 257.056	90.37218 (208.349558609609) 3, 13, 35, 92, 337	98.51862 (210.112588102877) 3, 15, 41, 103, 368	56.4387 (130.649164660148) 8, 17, 30, 55, 169	97.2606 (163.412654104725) 4, 17, 46, 112, 353	107.59848 (168.878932325987) 4, 21, 54, 127, 385
0.25	78.94764 (463.72954605057) 5, 12, 24, 50, 233.056	87.70244 (206.75562506451) 3, 12, 33, 87, 331	93.30164 (200.390387043528) 3, 14, 38, 97, 348	51.22158 (119.10316298727) 8, 16, 28, 51, 151	90.884 (157.088110681523) 3, 16, 43, 104, 327	103.5658 (161.840861217578) 4, 20, 52, 121, 371
0.5	58.22572 (301.00063654097) 5, 11, 20, 40, 173	75.87148 (223.227002955526) 2, 10, 27, 73, 287	82.46294 (175.511970861234) 2, 12, 33, 84, 311	39.37876 (90.7848219612748) 7, 14, 23, 40, 109	76.30304 (141.609807547042) 3, 14, 36, 86, 273	89.88488 (148.44741237243) 3, 17, 44, 104, 320
0.75	42.64318 (321.87917687166) 4, 9, 15, 29, 107	60.2345 (153.104376451987) 2, 8, 21, 56, 228	67.68566 (173.459201578517) 2, 9, 26, 67, 252	26.81558 (50.9129646736766) 6, 11, 18, 29, 70	58.9083 (110.964922725118) 3, 11, 27, 64, 212	72.14862 (119.488915653775) 3, 14, 35, 83, 259.056
1	28.35988 (312.032742536795) 3, 7, 12, 21, 64	45.59542 (143.009066589801) 2, 6, 15, 40, 171	53.7253 (136.464244876497) 2, 7, 19, 50, 206	18.48678 (25.2754062708168) 5, 9, 14, 21, 45	41.4341 (80.5815326539406) 2, 8, 19, 45, 151	55.07516 (100.347948584862) 2, 10, 26, 62, 197
1.5	11.75068 (110.81595825092) 3, 5, 7, 12, 27	22.97648 (88.1097951054789) 1, 3, 7, 18, 81	29.3634 (78.4786317308907) 1, 4, 10, 26, 109	10.23918 (7.81250239241388) 4, 6, 9, 12, 22	17.86684 (34.0538441545214) 1, 4, 9, 19, 62	27.9359 (49.7107040210847) 1, 5, 13, 31, 100
2	6.48688 (11.117072440692) 2, 3, 5, 7, 15	11.01084 (44.285979581061) 1, 2, 4, 9, 35	15.6834 (52.6900272273547) 1, 2, 5, 13, 55	6.66472 (3.6714161991714) 3, 4, 6, 8, 13	***8.12964 (16.2717045461678) 1, 3, 5, 9, 25	***13.64296 (25.2708134953882) 1, 3, 7, 15, 47
3	3.52292 (2.16039070988569) 2, 2, 3, 4, 7	3.38474 (13.0255237291162) 1, 1, 2, 3, 9	4.8707 (13.8546389534484) 1, 1, 2, 4, 15	3.89086 (1.58855875899465) 2, 3, 4, 5, 7	***2.86644 (2.64928331826591) 1, 1, 2, 3, 7	***4.13418 (6.59956413807278) 1, 1, 2, 5, 12
δ = 1.75						
0	39.78338 (169.340589760643) 4, 9, 17, 32, 114	52.0709 (106.494751494296) 2, 8, 22, 55, 193	58.7365 (115.46596570217) 2, 9, 25, 63, 220	29.38656 (45.7541727086482) 6, 12, 20, 33, 78	54.56972 (88.3724245046488) 3, 11, 27, 63, 193	65.49354 (98.4582670925889) 3, 13, 34, 78, 234
0.25	38.68182 (178.986255179213) 4, 9, 16, 31, 111	51.40096 (111.572987240228) 2, 8, 21, 52, 192	57.59702 (116.835385552777) 2, 9, 25, 61, 212	27.78528 (43.938886511114) 6, 12, 19, 31, 73	51.35624 (85.8896105100394) 2, 10, 25, 59, 183	62.33746 (96.6532262144992) 3, 12, 32, 73.25, 219
0.5	32.54756 (189.614221598981) 4, 8, 14, 27, 88	45.21512 (103.648204530477) 2, 7, 18, 46, 169	51.0938 (108.880620228529) 2, 8, 21, 53, 189	23.9906 (51.2001927812271) 5, 10, 17, 27, 62	45.39452 (76.1486961572431) 2, 9, 22, 52, 161	56.13328 (89.5879493930692) 2, 11, 28, 66, 197
0.75	25.20346 (118.638751034783) 3, 7, 12, 21, 64	38.27756 (93.7536881169328) 2, 6, 15, 38, 144	44.90748 (109.728507357733) 2, 7, 18, 45, 167	18.97884 (23.0119460986607) 5, 9, 14, 22, 47	36.59808 (66.329565146606) 2, 7, 18, 41, 129.056	46.75794 (74.0838608289627) 2, 9, 24, 55, 166
1	18.23442 (80.663661705814) 3, 6, 10, 17, 45	30.5454 (90.069233527196) 1, 5, 11, 28, 113	36.28098 (97.7680170707315) 1, 5, 14, 36, 131	14.63054 (13.9919789457352) 4, 8, 11, 17, 34	27.23666 (47.6748255353304) 2, 6, 13, 30, 96	37.0903 (62.1166869332703) 2, 7, 18, 42, 131.056
1.5	10.57668 (113.332991169894) 2, 4, 7, 11, 23	17.08192 (55.3435805432911) 1, 3, 6, 15, 59	21.48898 (52.9359583163775) 1, 3, 8, 21, 79	9.3244 (6.02034629663366) 3, 6, 8, 11, 20	14.15352 (24.9329144829499) 1, 3, 7, 16, 47	20.8228 (33.9636357113165) 1, 4, 10, 24, 74
2	6.1959 (7.22158474406583) 2, 3, 5, 7, 14	9.05054 (32.4235854199721) 1, 2, 4, 8, 29	12.48626 (34.8213173457759) 1, 2, 5, 12, 44	6.51332 (3.47714025999639) 3, 4, 6, 8, 13	7.2849 (11.1399844692477) 1, 2, 4, 8, 22	11.46536 (18.7397284830617) 1, 3, 6, 13, 39
3	3.57622 (2.01837359976783) 2, 2, 3, 4, 7	3.22492 (6.19576135295533) 1, 1, 2, 3, 9	4.56776 (12.3271557042878) 1, 1, 2, 4, 14	3.94624 (1.67667709691573) 2, 3, 4, 5, 7	***2.92392 (2.6819909945997) 1, 1, 2, 4, 7	***4.10968 (5.71414589653314) 1, 1, 3, 5, 12
δ = 2						
0	23.36936 (73.8518626382754) 4, 7, 12, 22, 65	34.50216 (68.4083695861478) 2, 6, 15, 36, 129	39.45016 (78.5004338990355) 2, 7, 18, 43, 144	19.3197 (22.7221402562161) 5, 9, 14, 23, 47	33.7683 (52.0188405705614) 2, 7, 17, 39, 120	42.18394 (62.5031419924757) 2, 9, 22, 50, 147
0.25	22.93326 (161.480032950806) 3, 7, 12, 21, 59	33.0679 (73.5826328535963) 2, 6, 14, 34, 121	38.64006 (74.3934786859024) 2, 6, 17, 42, 142	18.46528 (21.9097712284521) 5, 9, 14, 22, 45	32.8161 (51.5742947498808) 2, 7, 17, 38, 115	41.34056 (61.5220721220731) 2, 9, 22, 50, 145
0.5	19.54518 (55.6885703093989) 3, 7, 11, 19, 52	30.69984 (69.4903086914603) 1, 5, 13, 31, 113	34.86266 (66.5928381272806) 1, 6, 15, 38, 127	16.82014 (19.1146053514886) 4, 8, 13, 20, 41	28.78146 (44.4998169090896) 2, 6, 15, 33, 101	37.31864 (58.4311190416874) 2, 8, 19, 44, 129
0.75	17.0692 (111.085556988204) 3, 6, 10, 16, 42	26.32312 (58.6270761350651) 1, 4, 11, 26, 97	31.44788 (64.9520082735952) 1, 5, 13, 33, 116	14.41188 (13.8846192068278) 4, 8, 11, 17, 34	24.33226 (39.9744976516485) 2, 5, 12, 28, 85	32.59848 (49.0230970764857) 2, 7, 17, 38, 114.056
1	13.43172 (51.1155067835806) 3, 5, 9, 14, 34	21.21806 (48.896961951579) 1, 4, 9, 21, 77	25.85252 (52.6335472437067) 1, 4, 11, 27, 94	12.15918 (9.5456117719967) 4, 7, 10, 15, 27	19.48148 (34.2044508200134) 2, 4, 10, 22, 66	26.85314 (42.9349876022296) 1, 6, 14, 31, 93.0556
1.5	8.39936 (10.8404899296498) 2, 4, 6, 10, 20	13.18358 (34.5918945761432) 1, 3, 6, 13, 46	16.85546 (39.5409483690055) 1, 3, 7, 17, 60	8.4899 (5.28099382388536) 3, 5, 7, 10, 18	11.16134 (16.5692353533772) 1, 3, 6, 13, 37	16.52486 (26.3645664441438) 1, 4, 9, 19, 57
2	5.99486 (9.46120625865843) 2, 3, 5, 7, 13	7.69382 (19.855835386394) 1, 2, 4, 8, 25	10.40892 (25.6130619999347) 1, 2, 5, 10, 36	6.28228 (3.35785400297126) 2, 4, 6, 8, 12	6.49968 (8.76232705764338) 1, 2, 4, 8, 19	9.73636 (14.6373569675259) 1, 2, 5, 11, 32
3	3.6234 (2.17512001115829) 2, 2, 3, 4, 7	3.24568 (5.89681072997393) 1, 1, 2, 4, 9	4.30948 (9.31793961182257) 1, 1, 2, 4, 13	3.9916 (1.70573961405386) 2, 3, 4, 5, 7	***2.9498 (2.62896523163844) 1, 1, 2, 4, 7	***4.02556 (5.36626896672655) 1, 1, 3, 5, 12

LP-FK chart Lognormal distribution OOC performance

θ	m=50, n=5			m=100, n=5		
	MW-FK CUSUM chart			MW-FK CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
δ = 0.5						
-3	1.99816 (0.0428561679694144) 2, 2, 2, 2, 2	1.00016 (0.0126482251542258) 1, 1, 1, 1, 1	1.00008 (0.00894400357244998) 1, 1, 1, 1, 1	2.00108 (0.0328459309063321) 2, 2, 2, 2, 2	1.0004 (0.0199961995629154) 1, 1, 1, 1, 1	1.00032 (0.0178858602834743) 1, 1, 1, 1, 1
-2	2.14258 (0.353119579560167) 2, 2, 2, 2, 3	***1.18846 (0.396769425383986) 1, 1, 1, 1, 2	1.21194 (0.726382811635697) 1, 1, 1, 1, 2	2.61194 (0.493755316210616) 2, 2, 3, 3, 3	1.3853 (0.488926058800121) 1, 1, 1, 2, 2	1.35558 (0.541358222413311) 1, 1, 1, 2, 2
-1.5	2.89204 (0.650502191742679) 2, 2, 3, 3, 4	***1.95778 (1.21769746954757) 1, 1, 2, 2, 3	16.28354 (146.438006360584) 1, 1, 2, 4, 23	3.42296 (0.609222669838565) 3, 3, 3, 4, 4	2.19584 (0.709913215777449) 1, 2, 2, 2, 3	5.54112 (43.5005866071517) 1, 2, 3, 4, 11
-1	4.75238 (1.34891835453987) 3, 4, 5, 5, 7	***53.21308 (317.321995616269) 2, 3, 5, 10, 73	825.0112 (1181.53974519605) 2, 16, 111, 1357, 3001	5.85614 (1.34594224675899) 4, 5, 6, 7, 8	20.47486 (152.515205883679) 2, 4, 6, 10, 30	1170.44632 (1287.12472247233) 6, 52, 397, 3001, 3001
-0.75	6.82458 (13.7002540765796) 4, 5, 6, 8, 11	530.10736 (1026.06747123322) 3, 8, 25, 254, 3001	1991.24934 (1271.23670125771) 19, 447, 3001, 3001, 3001	8.45668 (2.17962804009488) 6, 7, 8, 10, 12	594.68662 (1054.96410348323) 5, 14, 45, 422, 3001	2650.43972 (850.045188365164) 272, 3001, 3001, 3001, 3001
-0.5	12.47122 (56.3953049492598) 5, 7, 9, 12, 22	1770.54976 (1348.00173258693) 11, 151, 3001, 3001, 3001	2798.0906 (665.274416788014) 740, 3001, 3001, 3001, 3001	13.04212 (5.6454533324142) 8, 10, 12, 15, 22	2483.83846 (1025.82423125597) 77, 3001, 3001, 3001, 3001	2986.40668 (176.979001699971) 3001, 3001, 3001, 3001, 3001
-0.25	62.11562 (304.525220965719) 7, 10, 15, 24, 100	2590.1324 (925.657471652088) 142, 3001, 3001, 3001, 3001	2975.83892 (231.832671772094) 3001, 3001, 3001, 3001, 3001	30.5133 (118.005630245702) 10, 14, 19, 27, 56	2963.29818 (288.877428803403) 3001, 3001, 3001, 3001, 3001	3000.26116 (36.9791355305082) 3001, 3001, 3001, 3001, 3001
0	257.6867 (685.567352223688) 9, 17, 31, 85, 2570	2653.40266 (831.818585466871) 330, 3001, 3001, 3001, 3001	2836.78874 (587.318868297515) 1147.95, 3001, 3001, 3001, 3001	183.38406 (526.442128276418) 14, 25, 40, 82, 825	2944.35228 (343.051500660533) 3001, 3001, 3001, 3001, 3001	2962.8032 (280.496191403783) 3001, 3001, 3001, 3001, 3001
0.25	255.71894 (643.417057479419) 11, 24, 45, 115, 1882.05	1851.71636 (1257.8871820921) 29, 414.75, 2730, 3001, 3001	1924.16878 (1249.79254896227) 35, 479, 3001, 3001, 3001	269.42392 (596.68503721185) 21, 42, 75, 167, 1500	2246.30508 (1104.3863982649) 113, 1252.75, 3001, 3001, 3001	2245.11954 (1099.91650156564) 121, 1248, 3001, 3001, 3001
0.5	61.75036 (256.882711396502) 4, 11, 20, 37, 141	551.82272 (944.19328139468) 3, 17, 86, 508, 3001	577.37718 (957.082685884872) 3, 21, 101, 558.25, 3001	45.74706 (139.731270866322) 7, 15, 25, 43, 111.056	523.28204 (871.001655435346) 4, 28, 118, 508, 3001	560.20624 (880.168899274956) 6, 39, 149, 586, 3001
0.75	10.15258 (39.15700965053) 2, 4, 6, 11, 26	62.68796 (277.607493395705) 1, 2, 6, 22, 212	67.66406 (278.988457566806) 1, 3, 8, 29, 238.049999999996	8.94372 (8.60961759150204) 3, 5, 7, 11, 21	27.23992 (118.426228403277) 1, 3, 7, 17, 92	39.062 (142.457560975659) 1, 4, 10, 28, 138
1	3.60994 (2.63502790570447) 1, 2, 3, 4, 8	4.52894 (24.936747969701) 1, 1, 2, 3, 12	6.42898 (41.1133511442749) 1, 1, 2, 4, 19	3.7373 (1.82930468679024) 2, 3, 3, 4, 7	2.92962 (4.94708764114025) 1, 1, 2, 3, 7	3.96822 (7.60418611716797) 1, 1, 2, 4, 12
1.5	1.4715 (0.598560703269359) 1, 1, 1, 2, 2	1.09006 (0.338808931744493) 1, 1, 1, 1, 2	1.1028 (0.445641259223872) 1, 1, 1, 1, 2	1.64004 (0.568467467411487) 1, 1, 2, 2, 2	1.07842 (0.283393559980104) 1, 1, 1, 1, 2	1.08098 (0.304867345712803) 1, 1, 1, 1, 2
2	1.03846 (0.19303257142636) 1, 1, 1, 1, 1	1.001 (0.0322338452626367) 1, 1, 1, 1, 1	1.0011 (0.0331483329330379) 1, 1, 1, 1, 1	1.0654 (0.247232810284658) 1, 1, 1, 1, 2	1.0005 (0.0223553124603505) 1, 1, 1, 1, 1	1.00034 (0.0184361383652856) 1, 1, 1, 1, 1
3	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1
δ = 1						
-3	***2.01988 (0.150084096774849) 2, 2, 2, 2, 2	1.06006 (0.239027067669713) 1, 1, 1, 1, 2	1.05774 (0.24332545396142) 1, 1, 1, 1, 2	***2.16848 (0.37498973578758) 2, 2, 2, 2, 3	***1.11572 (0.321140069469514) 1, 1, 1, 1, 2	***1.10234 (0.311878934475215) 1, 1, 1, 1, 2
-2	***2.6514 (0.659277432447569) 2, 2, 3, 3, 4	***1.68888 (0.803328856910448) 1, 1, 2, 2, 3	2.01812 (1.61188201456328) 1, 1, 2, 2, 5	***3.13706 (0.658971350868022) 2, 3, 3, 3, 4	***1.88726 (0.787935345851539) 1, 1, 2, 2, 3	2.27978 (1.5183903522432) 1, 1, 2, 3, 5
-1.5	***3.7849 (1.20722041799447) 2, 3, 4, 4, 6	3.2246 (2.5702853863607) 1, 2, 3, 4, 8	6.75004 (12.5795518561234) 1, 2, 4, 8, 21	4.47712 (1.22738202485318) 3, 4, 4, 5, 7	3.47154 (2.14100016488419) 1, 2, 3, 4, 7	7.83404 (10.3463828573644) 1, 3, 5, 9, 24
-1	***7.31976 (3.72598862078564) 3, 5, 7, 9, 14	21.23568 (71.933867854888) 2, 4, 9, 20, 71	74.08848 (200.011809115775) 2, 8, 23, 63, 279	8.60578 (3.32464007920153) 4, 6, 8, 10, 15	20.89484 (37.8559485792029) 2, 6, 11, 23, 68	133.70114 (411.067825398488) 3, 17, 47, 124, 481.056
-0.75	***13.81906 (16.4350139309359) 5, 7, 11, 16, 31	125.35966 (407.748567529759) 3, 12, 33, 96, 483	283.87892 (545.239788265831) 5, 27, 84, 257, 1358.09999999999	15.16784 (8.30211348008233) 6, 10, 13, 18, 30	158.5463 (461.449039787317) 5, 20, 55, 144, 598	858.42158 (2779.81416680798) 12, 77, 238, 705, 3281
-0.5	***48.2727 (176.789108212259) 7, 14, 22, 41, 139	568.58384 (1641.22021890025) 9, 52, 169, 502, 2232.05	693.21862 (878.521937848567) 14, 95, 305, 895, 3001	42.17724 (93.2471398304749) 11, 19, 29, 46, 107	1104.55458 (2288.52624234643) 22, 138, 416, 1137, 4294	2541.02374 (5376.64079156312) 50, 328, 977, 2651, 9726
-0.25	194.72104 (573.112807069525) 13, 31, 65, 162, 719	762.1248 (1805.88894623193) 16, 97, 287, 765, 2923	707.13618 (854.35064596191) 19, 117, 346, 922, 3001	219.83306 (493.903323617991) 22, 49, 96, 213, 781.056	1232.45186 (2119.83939165765) 34, 206, 573, 1401, 4497	1479.31376 (2750.28944974812) 37, 232, 651, 1616, 5513.15
0	239.32296 (864.494721973087) 11, 34, 81, 205, 857	308.95182 (967.24628473114) 6, 36, 107, 290, 1165	284.722 (490.21568671799) 6, 37, 109, 295, 1197.05	289.27622 (909.947959670386) 19, 53, 119, 281, 1014.05	309.42122 (549.526759882454) 9, 51, 139, 344, 1150	321.48444 (608.062368013968) 9.950027, 53, 146, 356, 1166
0.25	80.244 (272.97219195433) 5, 13, 28, 65, 279	70.9779 (154.624896924024) 2, 10, 28, 71, 267	72.9139 (153.55339690057) 2, 11, 29, 74, 271	63.6598 (132.742408577292) 7, 18, 33, 65, 207	65.03214 (116.661716368917) 3, 13, 33, 75, 227	68.6171 (109.787215249512) 3, 14, 35, 81, 241
0.5	17.77218 (39.4796860544739) 3, 6, 10, 19, 51	18.88384 (39.3518477102761) 1, 4, 9, 21, 66	20.16298 (36.931928962175) 1, 4, 10, 22, 71	15.34032 (14.2335011205495) 3, 7, 12, 19, 39	16.89674 (23.373641648551) 1, 4, 10, 21, 55.0556	19.1747 (26.0331791074062) 1, 5, 11, 24, 64
0.75	6.79112 (6.18423916435004) 2, 3, 5, 8, 17	6.54756 (9.46663458626178) 1, 2, 4, 8, 21	7.28344 (10.5634148588924) 1, 2, 4, 9, 24	6.86788 (4.47291230011152) 2, 4, 6, 9, 15	6.0329 (6.50660159432604) 1, 2, 4, 8, 18	6.92204 (7.89388678075447) 1, 2, 4, 9, 21
1	3.77664 (2.45162609706505) 1, 2, 3, 5, 8	3.06402 (3.33812287084913) 1, 1, 2, 4, 8	3.4005 (3.67866421427779) 1, 1, 2, 4, 10	3.97748 (2.16062634801215) 1, 2, 4, 5, 8	2.94298 (2.47719426961815) 1, 1, 2, 4, 8	3.28234 (3.08306571942354) 1, 1, 2, 4, 9
1.5	1.8895 (0.891058712831312) 1, 1, 2, 2, 3	1.39208 (0.743965283657017) 1, 1, 1, 2, 3	1.43802 (0.86854681340233) 1, 1, 1, 2, 3	2.02108 (0.870798943164258) 1, 1, 2, 2, 4	1.38806 (0.686060383623045) 1, 1, 1, 2, 3	1.40888 (0.768862125858318) 1, 1, 1, 2, 3
2	1.29248 (0.492646225608268) 1, 1, 1, 2, 2	1.0691 (0.269159133103038) 1, 1, 1, 1, 2	1.06848 (0.277113740981043) 1, 1, 1, 1, 2	1.35776 (0.519550941740674) 1, 1, 1, 2, 2	1.0649 (0.257467115931949) 1, 1, 1, 1, 2	1.06254 (0.256105564565091) 1, 1, 1, 1, 2
3	1.00866 (0.0926562253899978) 1, 1, 1, 1, 1	1.00024 (0.0154902291443341) 1, 1, 1, 1, 1	1.00038 (0.0194900794576098) 1, 1, 1, 1, 1	1.01142 (0.106253514747456) 1, 1, 1, 1, 1	1.00022 (0.0148309136306561) 1, 1, 1, 1, 1	1.0003 (0.0173180829862882) 1, 1, 1, 1, 1

δ = 1.25						
-3	***2.10786 (0.317975223193335) 2, 2, 2, 2, 3	***1.17268 (0.394087203160003) 1, 1, 1, 1, 2	***1.1797 (0.447495156441163) 1, 1, 1, 1, 2	***2.38012 (0.499993785437089) 2, 2, 2, 3, 3	***1.26178 (0.46014722242922) 1, 1, 1, 2, 2	***1.265 (0.517513629199394) 1, 1, 1, 1, 2
-2	3.00348 (0.877156358732896) 2, 2, 3, 3, 5	2.05452 (1.15933362354803) 1, 1, 2, 3, 4	2.63298 (2.31349591848944) 1, 1, 2, 3, 7	3.5382 (0.895118307675413) 2, 3, 3, 4, 5	2.27382 (1.15054295121115) 1, 2, 2, 3, 4	2.99758 (2.33884662765025) 1, 1, 2, 4, 7
-1.5	4.39722 (1.64704903600668) 2, 3, 4, 5, 7	4.1677 (3.69520362099249) 1, 2, 3, 5, 11	7.42784 (10.2252552071467) 1, 2, 4, 9, 23	5.20436 (1.68252596065326) 3, 4, 5, 6, 8	4.49806 (3.2483791767196) 1, 2, 4, 6, 11	9.4862 (10.8254049969443) 1, 3, 6, 12, 29
-1	8.57392 (4.67644450584676) 4, 5, 8, 10, 17	19.32104 (32.4413881051912) 2, 5, 10, 22, 65	40.72014 (76.8342773018903) 2, 7, 19, 45, 144	9.99906 (4.3356055016458) 5, 7, 9, 12, 18	22.07528 (28.5286797912734) 2, 6, 13, 27, 70	67.6931 (122.668226620696) 3, 13, 34, 77, 235
-0.75	14.6354 (11.8645590813668) 5, 8, 12, 18, 33	57.02066 (104.002847292799) 3, 10, 26, 63, 206	103.33742 (193.01594491348) 3, 17, 46, 114, 381	16.5122 (9.06295834174902) 7, 10, 14, 20, 33	77.55398 (122.724534924314) 4, 16, 40, 90, 271	198.07484 (326.731152147894) 7, 36, 97, 229, 712
-0.5	29.1573 (37.0927382938316) 7, 12, 20, 33, 79	124.51134 (203.172642090057) 5, 23, 61, 145, 444	175.87704 (279.758306713209) 6, 32, 87, 208, 629	31.02138 (26.165161860946) 10, 17, 25, 38, 72	195.67088 (270.592419489488) 8, 42, 106, 242, 678	318.7005 (446.738389511718) 12, 67, 176, 393, 1102
-0.25	***49.24272 (72.4411546113066) 8, 17, 30, 55, 150	123.709 (194.580870368344) 5, 24, 63, 147, 432	147.27384 (231.631013878078) 5, 28, 74, 175, 519	51.09864 (49.4520019833414) 11, 23, 37, 62, 137	161.64356 (221.793866012806) 7, 35, 91, 202, 548	191.01144 (270.842472411591) 7, 41, 105, 237.25, 652.056
0	43.23182 (62.0180961040605) 6, 14, 25, 49, 136	62.26288 (100.349104495063) 3, 12, 31, 72, 225	68.15494 (114.913290244784) 3, 13, 33, 79, 242	42.03202 (46.0143992812948) 7, 17, 29, 50, 117	62.9302 (84.233573792515) 3, 14, 36, 79, 213.056	69.19076 (93.6955936418984) 3, 16, 39, 86, 238
0.25	22.02258 (34.0084383884534) 3, 8, 14, 25, 64	24.08014 (36.353139459961) 1, 5, 13, 29, 83	26.37588 (39.2169962348147) 1, 6, 14, 32, 91	20.61482 (18.9042916770895) 4, 10, 16, 26, 53	23.03986 (29.3720779918567) 2, 6, 14, 29, 74	25.55994 (31.3634098985746) 2, 6, 15, 33, 83
0.5	10.33606 (10.3482919082962) 2, 5, 8, 13, 27	10.1871 (13.6327051828686) 1, 3, 6, 12, 33	11.35922 (16.1482617119839) 1, 3, 7, 14, 37	10.234 (7.27114993604367) 3, 5, 8, 13, 24	9.56742 (10.630884951776) 1, 3, 6, 12, 29	10.89974 (12.1518920873264) 1, 3, 7, 14, 34
0.75	5.75198 (4.38468363609186) 1, 3, 5, 7, 14	5.05592 (5.57883997106105) 1, 2, 3, 6, 15	5.5651 (6.3585509837865) 1, 2, 4, 7, 17	5.9674 (3.65495340786832) 2, 3, 5, 8, 13	4.7962 (4.60774241747558) 1, 2, 3, 6, 13	5.37034 (5.39829705709331) 1, 2, 4, 7, 16
1	3.72312 (2.31683508694434) 1, 2, 3, 5, 8	2.9774 (2.69734587205554) 1, 1, 2, 4, 8	3.21706 (3.09300117839062) 1, 1, 2, 4, 9	3.92366 (2.14181323921556) 1, 2, 3, 5, 8	2.85254 (2.25989771696726) 1, 1, 2, 4, 7	3.11986 (2.7520546972219) 1, 1, 2, 4, 8
1.5	2.0729 (1.02473732816358) 1, 1, 2, 3, 4	1.54224 (0.909303205232656) 1, 1, 1, 2, 3	1.59142 (1.03604239839739) 1, 1, 1, 2, 4	2.18906 (1.01027557108672) 1, 1, 2, 3, 4	1.52146 (0.824495642354333) 1, 1, 1, 2, 3	1.5507 (0.936401114342) 1, 1, 1, 2, 3
2	1.43954 (0.60069277092046) 1, 1, 1, 2, 2	1.14576 (0.398894979592492) 1, 1, 1, 1, 2	1.14718 (0.415934499206489) 1, 1, 1, 1, 2	1.51568 (0.616313016603324) 1, 1, 1, 2, 3	1.13958 (0.381864295276215) 1, 1, 1, 1, 2	1.13824 (0.394122834984451) 1, 1, 1, 1, 2
3	1.0417 (0.200603875431899) 1, 1, 1, 1, 1	1.00432 (0.0655852394100423) 1, 1, 1, 1, 1	1.00422 (0.0654391109776478) 1, 1, 1, 1, 1	1.05418 (0.22655143811303) 1, 1, 1, 1, 2	1.0031 (0.0555918322151391) 1, 1, 1, 1, 1	1.00342 (0.0587228453623575) 1, 1, 1, 1, 1
δ = 1.5						
-3	2.26348 (0.474323506961004) 2, 2, 2, 3, 3	1.31762 (0.541094623210973) 1, 1, 1, 2, 2	1.36914 (0.692766381188621) 1, 1, 1, 2, 3	2.6024 (0.586038487492227) 2, 2, 3, 3, 3	1.42856 (0.591796697242066) 1, 1, 1, 2, 2	1.48826 (0.759100584292979) 1, 1, 1, 2, 3
-2	3.38846 (1.12284640271093) 2, 3, 3, 4, 5	2.48922 (1.62596945455114) 1, 1, 2, 3, 6	3.29006 (2.99962416834226) 1, 1, 2, 4, 9	3.9811 (1.16238109611102) 2, 3, 4, 5, 6	2.73326 (1.59862468528973) 1, 2, 2, 3, 6	3.84362 (3.27825871999217) 1, 2, 3, 5, 10
-1.5	4.97932 (2.04725576387594) 2, 4, 5, 6, 9	5.03442 (4.66923455168253) 1, 2, 4, 6, 14	8.23248 (9.68156122235221) 1, 2, 5, 10, 26	5.8446 (2.06967062856151) 3, 4, 6, 7, 10	5.5018 (4.50748744966219) 1, 3, 4, 7, 14	10.71714 (11.6181302213134) 1, 3, 7, 14, 33
-1	9.01506 (4.91033557145624) 4, 6, 8, 11, 18	17.32468 (22.7324111820098) 2, 5, 10, 21, 56	29.41444 (41.4532580979896) 2, 7, 16, 36, 101	10.41194 (4.63772741832809) 5, 7, 10, 13, 19	20.42954 (24.0553047900493) 2, 6, 13, 26, 64	45.6431 (58.8319643254458) 2, 11, 27, 58, 152
-0.75	13.15844 (8.95202432644097) 4, 8, 11, 16, 29	34.13008 (46.6979105859209) 2, 8, 19, 42, 115	52.4849 (71.7989392243609) 2, 12, 29, 65, 178	14.73932 (7.53125888114376) 6, 9, 13, 18, 29	43.3324 (52.9610298960826) 3, 11, 26, 55, 141	84.76584 (107.798020841574) 4, 20, 50, 109, 281
-0.5	18.574 (14.9081510945986) 5, 10, 15, 23, 45	48.7671 (63.1439409661419) 3, 11, 28, 61, 165	65.09574 (84.0086440962956) 3, 15, 38, 83, 218	20.18678 (12.088805400697) 7, 12, 17, 25, 43	62.60296 (75.9338751698237) 4, 16, 38, 81, 205	93.93256 (112.854631695843) 5, 23, 57, 122, 305.056
-0.25	21.48832 (19.7295648378816) 5, 10, 16, 26, 55	42.98624 (57.6172610043007) 2, 10, 24, 54, 145	51.01032 (68.5275014418786) 2, 11, 29, 64, 173	22.24486 (15.2187481675175) 6, 12, 19, 28, 50	46.8304 (57.1302196134398) 3, 12, 28, 60, 153	57.67572 (70.5543523874638) 3, 14, 35, 75, 184
0	17.89694 (17.3168783596214) 4, 8, 13, 22, 47	24.96084 (33.0616443623265) 2, 6, 14, 31, 83	28.24768 (37.5846147603369) 2, 7, 16, 35, 94	18.0414 (13.0207156800514) 4, 10, 15, 23, 42	24.37122 (28.2625991586376) 2, 7, 15, 32, 78	28.59394 (33.505534284547) 2, 8, 18, 37, 92
0.25	11.79876 (10.2912924687045) 2, 6, 9, 15, 30	13.04844 (16.0701903718273) 1, 4, 8, 16, 42	14.40942 (17.615342212663) 1, 4, 9, 18, 47	11.93104 (8.11252616513596) 3, 6, 10, 15, 27	12.48264 (13.8778467538379) 1, 4, 8, 16, 38	14.07522 (15.5497767785508) 1, 4, 9, 19, 44
0.5	7.56336 (5.72744983251985) 2, 4, 6, 10, 18	7.12148 (7.98746008440556) 1, 2, 5, 9, 22	7.96184 (9.01058752957611) 1, 2, 5, 10, 25	7.7429 (4.88076182851822) 2, 4, 7, 10, 17	6.75442 (6.67001351162087) 1, 2, 5, 9, 20	7.60522 (7.84852347502476) 1, 2, 5, 10, 23
0.75	5.0323 (3.31770957950467) 1, 3, 4, 6, 11	4.31572 (4.28446589606689) 1, 2, 3, 5, 12	4.75918 (4.84069978154889) 1, 2, 3, 6, 14	5.2682 (3.07320315819717) 2, 3, 5, 7, 11	4.08334 (3.61111274361587) 1, 2, 3, 5, 11	4.57646 (4.34623189245963) 1, 2, 3, 6, 13
1	3.61666 (2.1714706422365) 1, 2, 3, 5, 8	2.84612 (2.44564931451985) 1, 1, 2, 4, 7	3.10934 (2.84712958738055) 1, 1, 2, 4, 9	3.8125 (2.07163451974321) 1, 2, 3, 5, 8	2.76166 (2.14397434147882) 1, 1, 2, 4, 7	2.99618 (2.50822073012417) 1, 1, 2, 4, 8
1.5	2.2053 (1.12560083934981) 1, 1, 2, 3, 4	1.6539 (1.02575622519031) 1, 1, 1, 2, 4	1.7047 (1.14643979191405) 1, 1, 1, 2, 4	2.31938 (1.10775491784923) 1, 2, 2, 3, 4	1.6195 (0.928125518633965) 1, 1, 1, 2, 3	1.6763 (1.07224125271605) 1, 1, 1, 2, 4
2	1.57448 (0.69912982002637) 1, 1, 1, 2, 3	1.22824 (0.506864519011553) 1, 1, 1, 1, 2	1.23694 (0.548165528019523) 1, 1, 1, 1, 2	1.6587 (0.705906704944444) 1, 1, 2, 2, 3	1.2179 (0.48351242557489) 1, 1, 1, 1, 2	1.22328 (0.517369688934229) 1, 1, 1, 1, 2
3	1.10108 (0.306504017093169) 1, 1, 1, 1, 2	1.01846 (0.135497585188489) 1, 1, 1, 1, 1	1.01724 (0.132752155774199) 1, 1, 1, 1, 1	1.12574 (0.334681479370838) 1, 1, 1, 1, 2	1.01518 (0.12389461084927) 1, 1, 1, 1, 1	1.01438 (0.120388975700059) 1, 1, 1, 1, 1

δ = 1.75						
-3	2.45022 (0.613391780666755) 2, 2, 2, 3, 4	1.49048 (0.700113685613598) 1, 1, 1, 2, 3	1.60518 (0.970461747582083) 1, 1, 1, 2, 3	2.83274 (0.691703449082936) 2, 2, 3, 3, 4	1.62784 (0.747762072695819) 1, 1, 1, 2, 3	1.7547 (1.0372605401675) 1, 1, 1, 2, 4
-2	3.75228 (1.34378231742285) 2, 3, 4, 4, 6	2.9451 (2.11328543020251) 1, 2, 2, 4, 7	3.98628 (3.70964243432434) 1, 1, 3, 5, 11	4.3908 (1.39467353251047) 3, 3, 4, 5, 7	3.20942 (2.07604659460939) 1, 2, 3, 4, 7	4.70702 (4.21427786487299) 1, 2, 3, 6, 13
-1.5	5.40978 (2.3215701896266) 3, 4, 5, 7, 10	5.75508 (5.42339031020718) 1, 2, 4, 7, 16	8.91842 (9.87116778819355) 1, 3, 6, 12, 27	6.31782 (2.34273349222747) 3, 5, 6, 8, 11	6.31956 (5.3741547276926) 1, 3, 5, 8, 17	11.45458 (12.1531399646272) 1, 4, 8, 15, 35
-1	8.68986 (4.63592741712598) 3, 6, 8, 11, 17	14.82372 (17.1239478495293) 2, 4, 9, 19, 47	23.11652 (28.4959467901888) 1, 6, 14, 30, 75	9.86132 (4.34124691604794) 4, 7, 9, 12, 18	16.6866 (18.0279061609498) 2, 5, 11, 21, 51	32.88386 (38.1733812445415) 2, 9, 21, 43, 106
-0.75	10.9677 (6.59183477915786) 4, 7, 9, 14, 23	22.14118 (26.3028972749334) 2, 6, 13, 28, 71	31.92186 (38.1215820663412) 2, 8, 20, 42, 104	12.16174 (5.84866004388066) 5, 8, 11, 15, 23	25.50486 (28.331672560119) 2, 7, 16, 33, 79	44.43522 (50.8923251986671) 3, 12, 28, 59, 142
-0.5	12.85954 (8.59545976821325) 4, 7, 11, 16, 29	25.37552 (29.7670211180179) 2, 7, 15, 33, 82	33.34908 (39.8458642397369) 2, 8, 21, 43, 108	13.82466 (7.41070673120217) 5, 9, 12, 17, 28	27.7079 (31.3607105702876) 2, 8, 17, 36, 87	41.0905 (46.914196237239) 2, 11, 26, 54, 131
-0.25	12.8803 (9.43936830214177) 3, 7, 11, 16, 30	20.87138 (25.1100969180715) 2, 5, 13, 27, 67	25.62182 (31.0691149807283) 2, 7, 16, 33, 83	13.47022 (7.85997129349559) 4, 8, 12, 17, 28	21.1551 (23.7191276384761) 2, 6, 14, 27, 66	27.1605 (30.7221629573169) 2, 7, 17, 36, 87
0	10.9763 (8.16387109684038) 3, 6, 9, 14, 26	14.00026 (16.4658750885961) 1, 4, 9, 18, 45	16.18988 (19.0874721274831) 1, 4, 10, 21, 52	11.32276 (6.79076934326041) 3, 7, 10, 14, 24	13.47264 (14.4543304924095) 1, 4, 9, 18, 41	16.0575 (17.4786104649791) 1, 5, 11, 21, 50
0.25	8.3261 (5.93082644190889) 2, 4, 7, 11, 19	8.75612 (9.7287314440224) 1, 3, 6, 11, 27	9.873 (10.9254719952397) 1, 3, 6, 13, 31	8.60726 (5.14034860258282) 2, 5, 8, 11, 18	8.28032 (8.40858102073683) 1, 3, 6, 11, 24	9.52162 (9.85706831727955) 1, 3, 6, 13, 29
0.5	6.1214 (4.10145321543103) 2, 3, 5, 8, 14	5.60084 (5.63538343374512) 1, 2, 4, 7, 16	6.25414 (6.59711022324669) 1, 2, 4, 8, 19	6.36322 (3.70620101318304) 2, 4, 6, 8, 13	5.29344 (4.88541198993557) 1, 2, 4, 7, 15	5.9608 (5.82651716946295) 1, 2, 4, 8, 17
0.75	4.5611 (2.8529405138193) 1, 3, 4, 6, 10	3.8215 (3.53265443299325) 1, 2, 3, 5, 11	4.20058 (4.06626097947083) 1, 1, 3, 5, 12	4.74866 (2.65417955266443) 1, 3, 4, 6, 10	3.6271 (3.07019121741338) 1, 2, 3, 5, 10	4.00594 (3.60437574995878) 1, 2, 3, 5, 11
1	3.501 (2.05452754220245) 1, 2, 3, 4, 7	2.75642 (2.26192641565153) 1, 1, 2, 3, 7	2.97982 (2.6133942228809) 1, 1, 2, 4, 8	3.66658 (1.96907304297554) 1, 2, 3, 5, 7	2.6548 (2.00353618463577) 1, 1, 2, 3, 7	2.85976 (2.35931432193368) 1, 1, 2, 4, 8
1.5	2.30124 (1.19324890079932) 1, 1, 2, 3, 5	1.73978 (1.09354902431915) 1, 1, 1, 2, 4	1.79684 (1.23584649561797) 1, 1, 1, 2, 4	2.40876 (1.17748163113478) 1, 2, 2, 3, 5	1.69832 (0.998423314322283) 1, 1, 1, 2, 4	1.7535 (1.14110638963939) 1, 1, 1, 2, 4
2	1.6989 (0.783154554747103) 1, 1, 2, 2, 3	1.31172 (0.609260260940521) 1, 1, 1, 1, 2	1.32184 (0.658989908689045) 1, 1, 1, 1, 3	1.77154 (0.784498780857398) 1, 1, 2, 2, 3	1.29174 (0.570187929270248) 1, 1, 1, 1, 2	1.29472 (0.606454843619496) 1, 1, 1, 1, 2
3	1.1694 (0.38949540964366) 1, 1, 1, 1, 2	1.04392 (0.209647114547882) 1, 1, 1, 1, 1	1.04216 (0.208766391136412) 1, 1, 1, 1, 1	1.20054 (0.412464678417703) 1, 1, 1, 1, 2	1.03674 (0.191705261842622) 1, 1, 1, 1, 1	1.03588 (0.190770420838948) 1, 1, 1, 1, 1
δ = 2						
-3	2.64966 (0.741008006878013) 2, 2, 3, 3, 4	1.68726 (0.881061415444097) 1, 1, 1, 2, 3	1.88704 (1.28697830776719) 1, 1, 1, 2, 4	3.07794 (0.808330640498359) 2, 3, 3, 4, 4	1.82484 (0.916698304304978) 1, 1, 2, 2, 4	2.072 (1.38887529299487) 1, 1, 2, 3, 5
-2	4.06302 (1.52185899501104) 2, 3, 4, 5, 7	3.38226 (2.56571022326743) 1, 2, 3, 4, 8	4.6739 (4.45655427516822) 1, 2, 3, 6, 13	4.75268 (1.5908373370237) 3, 4, 5, 6, 8	3.65286 (2.51841233089686) 1, 2, 3, 5, 9	5.48778 (5.08485671252944) 1, 2, 4, 7, 15
-1.5	5.61706 (2.45101960545854) 3, 4, 5, 7, 10	6.15116 (5.78188025275387) 1, 2, 4, 8, 17	9.40018 (10.0670265157124) 1, 3, 6, 12, 29	6.47202 (2.43851103059242) 3, 5, 6, 8, 11	6.56418 (5.5808945389854) 1, 3, 5, 8, 17	11.87146 (12.352447090653) 1, 4, 8, 16, 36
-1	7.9227 (4.03091176720522) 3, 5, 7, 10, 15	12.16774 (13.2594373766759) 1, 4, 8, 16, 37	18.47138 (20.7510952254251) 1, 5, 12, 24, 58	8.89258 (3.84004373182354) 4, 6, 8, 11, 16	13.14322 (13.3754882567076) 2, 4, 9, 17, 39	24.02884 (26.5721291923586) 2, 7, 16, 32, 75
-0.75	9.04816 (5.02997680190799) 3, 6, 8, 11, 18	15.20144 (17.2257016235139) 1, 4, 10, 19, 48	21.6518 (24.56193442163) 1, 6, 14, 28, 68	9.9412 (4.62538327529826) 4, 7, 9, 12, 19	15.87082 (16.8522607534256) 2, 5, 10, 21, 48	26.33432 (28.8363655972319) 2, 7, 17, 35, 81
-0.5	9.60288 (5.73068517026109) 3, 6, 8, 12, 20	15.34854 (17.3826575357182) 1, 4, 10, 20, 48.0556	20.4792 (23.1802500814783) 1, 5, 13, 27, 64	10.25882 (5.11654920714301) 4, 7, 9, 13, 20	15.58356 (16.3410917123663) 2, 5, 10, 20, 47	22.53458 (24.5368752164677) 2, 6, 15, 30, 70
-0.25	***9.18946 (5.76749080514254) 3, 5, 8, 12, 20	12.8997 (14.3993592475823) 1, 4, 8, 17, 40	15.77236 (17.8030255570445) 1, 4, 10, 21, 50	9.67852 (5.12684857403903) 3, 6, 9, 12, 19	12.22162 (12.4893292150734) 1, 4, 8, 16, 36	15.99762 (17.0166061725984) 1, 5, 11, 21, 49
0	8.06762 (5.15811396330027) 2, 5, 7, 10, 18	***9.38008 (10.1603978203256) 1, 3, 6, 12, 29	11.1166 (12.4094707517801) 1, 3, 7, 14, 35	8.33762 (4.60127549385205) 3, 5, 8, 11, 17	8.78198 (8.76301906249563) 1, 3, 6, 11, 26	10.70288 (11.1694536499999) 1, 3, 7, 14, 32
0.25	6.60192 (4.20018632264128) 2, 4, 6, 8, 14	6.6055 (6.8209383704887) 1, 2, 4, 8, 19	7.53946 (7.89238168756264) 1, 2, 5, 10, 23	6.78158 (3.77960823530273) 2, 4, 6, 9, 14	6.1194 (5.69579076868608) 1, 2, 4, 8, 17	7.23698 (7.14488918517491) 1, 2, 5, 10, 21
0.5	5.24372 (3.21599866519691) 1, 3, 5, 7, 11	4.69086 (4.46465352776926) 1, 2, 3, 6, 13	5.25222 (5.17737203368886) 1, 2, 4, 7, 15	5.4191 (3.01976117750252) 2, 3, 5, 7, 11	4.38536 (3.75782651217455) 1, 2, 3, 6, 12	4.96758 (4.61994759947734) 1, 2, 4, 7, 14
0.75	4.20446 (2.49499912000228) 1, 2, 4, 5, 9	3.45242 (2.9829840944065) 1, 1, 3, 4, 9	3.81822 (3.53959130490119) 1, 1, 3, 5, 11	4.33204 (2.3495233226331) 1, 3, 4, 6, 9	3.25818 (2.61367551803859) 1, 1, 2, 4, 8	3.60626 (3.15375137500639) 1, 1, 3, 5, 10
1	3.38964 (1.93220478694852) 1, 2, 3, 4, 7	2.66144 (2.06129621967395) 1, 1, 2, 3, 7	2.87918 (2.46627333799065) 1, 1, 2, 4, 8	3.53946 (1.86560245439653) 1, 2, 3, 5, 7	2.55658 (1.87336832838016) 1, 1, 2, 3, 6	2.7329 (2.20771716265952) 1, 1, 2, 3, 7
1.5	2.36238 (1.22781549355664) 1, 1, 2, 3, 5	1.78352 (1.12923067299744) 1, 1, 1, 2, 4	1.8635 (1.3154703946677) 1, 1, 1, 2, 4	2.4692 (1.21452906989386) 1, 2, 2, 3, 5	1.73986 (1.04552811678159) 1, 1, 1, 2, 4	1.79726 (1.18697290217318) 1, 1, 1, 2, 4
2	1.7889 (0.850853259396343) 1, 1, 2, 2, 3	1.378 (0.685656913119127) 1, 1, 1, 2, 3	1.39058 (0.743154297011182) 1, 1, 1, 2, 3	1.86714 (0.846771846987851) 1, 1, 2, 2, 3	1.35286 (0.642088830272976) 1, 1, 1, 2, 3	1.3578 (0.686082046238993) 1, 1, 1, 2, 3
3	1.24182 (0.458112742614412) 1, 1, 1, 1, 2	1.07336 (0.271550704649618) 1, 1, 1, 1, 2	1.0711 (0.275184128405129) 1, 1, 1, 1, 2	1.28214 (0.482308690398024) 1, 1, 1, 2, 2	1.06338 (0.251722551403043) 1, 1, 1, 1, 2	1.06162 (0.254527545239068) 1, 1, 1, 1, 2

LP-SR chart Normal distribution OOC performance

	m=50, n=5			m=100, n=5		
	MW-SR CUSUM chart			MW-SR CUSUM chart		
θ	K=0	K=3	K=6	K=0	K=3	K=6
δ = 0.5						
0	83.20586 (440.668719947394) 9, 14, 20, 34, 131	2940.18184 (363.164987671416) 3001, 3001, 3001, 3001, 3001	2966.05218 (273.010763441273) 3001, 3001, 3001, 3001, 3001	***37.60678 (136.009272950333) 13, 19, 25, 36, 72	2723.88116 (749.688393007913) 508, 3001, 3001, 3001, 3001	2997.5731 (82.6017999932581) 3001, 3001, 3001, 3001, 3001
0.25	44.97576 (261.748491376507) 9, 13, 18, 28, 72	2719.48906 (766.733564236943) 421, 3001, 3001, 3001, 3001	2753.8117 (721.219444907626) 542.95, 3001, 3001, 3001, 3001	***29.67526 (69.7516711616646) 13, 18, 24, 32, 57	1711.7566 (1282.97841016459) 21, 286.75, 1927, 3001, 3001	2903.2744 (443.380514380925) 2407.8, 3001, 3001, 3001, 3001
0.5	16.16048 (28.1653786046032) 7, 10, 13, 18, 32	1696.6355 (1317.55358744488) 12, 204, 2023.5, 3001, 3001	1814.5717 (1293.41212777589) 19, 310, 2865, 3001, 3001	***19.2749 (7.78846731998433) 10, 14, 18, 22, 33	439.12626 (844.01470116031) 2, 13, 61, 322, 3001	1968.1211 (1211.77620474732) 55, 603, 3001, 3001, 3001
0.75	8.95554 (5.04547643390086) 4, 6, 8, 11, 17	445.9136 (900.088766939577) 2, 8, 33, 259, 3001	600.23478 (1006.86956725529) 2, 15, 80, 565, 3001	***10.6901 (4.15813994383977) 5, 8, 10, 13, 18	41.09684 (200.647701835878) 1, 2, 5, 16, 130	455.0616 (827.200878224107) 3, 20, 85, 388, 3001
1	5.14328 (2.4273913335459) 2, 3, 5, 6, 10	39.31958 (233.134884575672) 1, 2, 4, 10, 91	92.54362 (368.927436125803) 1, 2, 7, 27, 357	***5.69262 (2.27189805361405) 3, 4, 5, 7, 10	3.81122 (24.7490095232324) 1, 1, 2, 3, 10	33.3244 (140.289575810707) 1, 3, 7, 20, 113
1.5	2.19234 (0.843919171716917) 1, 2, 2, 3, 4	1.41232 (2.30281095106948) 1, 1, 1, 2, 3	1.7181 (5.18972552790811) 1, 1, 1, 2, 4	***2.34246 (0.71071179153368) 1, 2, 2, 3, 4	1.07988 (0.320283057973325) 1, 1, 1, 1, 2	1.3955 (0.863686485615321) 1, 1, 1, 2, 3
2	1.30886 (0.476686526976482) 1, 1, 1, 2, 2	1.01372 (0.120548962007331) 1, 1, 1, 1, 1	1.01494 (0.139703925298235) 1, 1, 1, 1, 1	***1.46422 (0.505811139159835) 1, 1, 1, 2, 2	1.00082 (0.0286241853487525) 1, 1, 1, 1, 1	1.0061 (0.0791417484115316) 1, 1, 1, 1, 1
3	1.00026 (0.0161225803999228) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	***1.00058 (0.0240764447853887) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1
δ = 1						
0	***527.69222 (2682.05774307083) 15, 45, 116, 346, 1850.05	487.6361 (1462.64773801654) 9, 53, 159, 442, 1871	498.511 (2372.75352071478) 9, 56, 167, 460, 1865	481.28624 (1399.81148877652) 25, 69, 162, 420, 1799	508.5979 (911.88951724725) 15, 84, 233, 562, 1878	498.33468 (853.681119563023) 15, 88, 239, 569, 1801
0.25	231.63786 (1483.91528244908) 9, 24, 53, 141, 792	***297.72628 (1019.64795779549) 5, 28, 84, 248, 1129.05	288.4365 (839.608343509734) 5, 30, 90, 255, 1104.05	163.2878 (536.587600915479) 14, 33, 63, 137, 555	***251.52876 (495.318125141574) 7, 39, 107, 269, 936.056	***257.38728 (506.376672172884) 7, 43, 116, 288, 937.056
0.5	37.95584 (132.947106882997) 5, 10, 18, 33, 109	80.22532 (305.005162306933) 2, 9, 24, 67, 304.056	82.99662 (225.09051149114) 2, 10, 28, 75, 314	28.61714 (32.1392877471909) 7, 13, 21, 34, 74	59.6307 (112.694431087814) 2, 11, 27, 66, 215	65.91812 (112.848071624689) 3, 12, 32, 75, 235
0.75	11.22992 (11.792032811791) 3, 5, 9, 13, 27	19.48684 (59.6625981610488) 1, 3, 8, 19, 68	22.60466 (51.0978463937818) 1, 4, 10, 23, 80	11.2786 (7.1828917518419) 4, 7, 10, 14, 24	***15.25748 (23.9811935900305) 1, 4, 8, 18, 51	19.14366 (30.4269646499278) 1, 4, 10, 23, 65
1	5.85154 (3.81830213799594) 2, 3, 5, 7, 13	6.3063 (10.8174757067263) 1, 2, 3, 7, 20	7.8187 (12.7493121857308) 1, 2, 4, 9, 26	6.22184 (3.16851192261542) 2, 4, 6, 8, 12	5.46408 (6.19465876619081) 1, 2, 4, 7, 16	6.94812 (8.29778557993099) 1, 2, 4, 9, 22
1.5	2.6788 (1.23504698972198) 1, 2, 2, 3, 5	1.86616 (1.34610664304946) 1, 1, 1, 2, 4	2.05818 (1.895844659285) 1, 1, 1, 2, 5	2.90116 (1.17798913728784) 1, 2, 3, 4, 5	1.82974 (1.11411685093044) 1, 1, 2, 2, 4	1.9831 (1.49657581992484) 1, 1, 1, 2, 5
2	1.70584 (0.676667608195515) 1, 1, 2, 2, 3	1.18038 (0.453836066155499) 1, 1, 1, 1, 2	1.1878 (0.502290957438135) 1, 1, 1, 1, 2	1.85872 (0.65979441356635) 1, 1, 2, 2, 3	1.18192 (0.428239163757389) 1, 1, 1, 1, 2	1.18212 (0.464108407090704) 1, 1, 1, 1, 2
3	1.07136 (0.258126098993611) 1, 1, 1, 1, 2	1.00232 (0.0481109539842837) 1, 1, 1, 1, 1	1.00234 (0.0483173994687403) 1, 1, 1, 1, 1	1.12722 (0.333642470237167) 1, 1, 1, 1, 2	1.00216 (0.0464260434190784) 1, 1, 1, 1, 1	1.00158 (0.0397181967201812) 1, 1, 1, 1, 1
δ = 1.25						
0	43.9264 (92.3498003936806) 6, 13, 24, 44, 134	60.77918 (106.912742669985) 3, 11, 29, 68, 219	66.79044 (112.686284385369) 3, 13, 33, 77, 236	37.5193 (39.7155497759741) 8, 17, 27, 44, 98	62.99874 (84.1878768611289) 3, 15, 36, 79, 213	71.68862 (94.4544140589645) 3, 17, 42, 90, 239
0.25	30.98592 (52.2573675035661) 5, 11, 19, 33, 91	44.77266 (80.1285825927263) 2, 8, 21, 50, 163	50.62622 (92.6051036531791) 2, 9, 24, 57, 181	28.2071 (26.5093791804654) 7, 13, 21, 34, 71	44.02284 (60.6040758977043) 2, 10, 25, 55, 147.056	51.08852 (66.870068014991) 2, 12, 29, 64, 171
0.5	16.25022 (19.748197127267) 3, 7, 12, 19, 42	21.96494 (37.4296017918733) 1, 5, 11, 25, 78	25.17162 (43.3608345147438) 1, 5, 13, 29, 89	15.8206 (11.2138312204265) 5, 9, 13, 20, 36	20.15144 (26.1645102678868) 2, 5, 12, 25, 65	24.54654 (31.7101886583744) 1, 6, 14, 31, 80
0.75	8.83842 (6.90649157648932) 2, 5, 7, 11, 21	9.84862 (14.6008673647204) 1, 3, 5, 11, 32	11.54414 (16.6464144404335) 1, 3, 6, 14, 38	9.26928 (5.49595236437215) 3, 5, 8, 12, 19	9.0706 (10.3236963929662) 1, 3, 6, 11, 27	11.15104 (13.0831361029153) 1, 3, 7, 14, 35
1	5.53304 (3.45193665581127) 2, 3, 5, 7, 12	4.98172 (5.70303395865102) 1, 2, 3, 6, 15	5.73548 (7.04043326249248) 1, 2, 4, 7, 18	5.91542 (3.04413067364579) 2, 4, 5, 7, 12	4.69072 (4.46051384991417) 1, 2, 3, 6, 13	5.66764 (5.98310728483583) 1, 2, 4, 7, 17
1.5	2.86714 (1.3850077925166) 1, 2, 3, 4, 5	2.01616 (1.44954505898146) 1, 1, 2, 2, 5	2.18962 (1.84819170322005) 1, 1, 2, 3, 6	3.12654 (1.35726359698086) 1, 2, 3, 4, 6	2.00346 (1.30381057935596) 1, 1, 2, 2, 4	2.1816 (1.65826307826253) 1, 1, 2, 3, 5
2	1.8913 (0.785605914904585) 1, 1, 2, 2, 3	1.306 (0.604128545448722) 1, 1, 1, 1, 2	1.32656 (0.696626350467339) 1, 1, 1, 1, 3	2.06982 (0.791073753548726) 1, 2, 2, 2, 3	1.30906 (0.586096226379023) 1, 1, 1, 2, 2	1.32868 (0.656153997443715) 1, 1, 1, 2, 3
3	1.18066 (0.389287805228864) 1, 1, 1, 1, 2	1.01438 (0.120720781434438) 1, 1, 1, 1, 1	1.01278 (0.113740627475879) 1, 1, 1, 1, 1	1.26534 (0.446742292573156) 1, 1, 1, 2, 2	1.0153 (0.123232356644976) 1, 1, 1, 1, 1	1.01556 (0.125531675540603) 1, 1, 1, 1, 1

δ = 1.5						
0	14.0481 (12.2754828882453) 4, 7, 11, 17, 34	17.72192 (23.628277072676) 1, 4, 10, 22, 58	20.85236 (27.3832116311434) 1, 5, 12, 26, 69	14.6691 (9.04226743919958) 5, 9, 13, 18, 31	18.09494 (20.8437188526595) 2, 5, 11, 23, 57	22.6716 (25.7154082037284) 2, 6, 14, 30, 71
0.25	12.57566 (10.3218857962549) 3, 6, 10, 15, 30	15.19058 (20.3750053271267) 1, 4, 9, 19, 50	18.02132 (23.810284417673) 1, 4, 11, 22, 60	13.10952 (7.88149019712355) 4, 8, 11, 16, 28	15.27948 (17.1730471728399) 1, 5, 10, 20, 48	18.94064 (21.4539525922856) 1, 5, 12, 25, 59
0.5	9.51856 (7.00434698911575) 3, 5, 8, 12, 22	10.3768 (13.2792013503738) 1, 3, 6, 13, 33	12.13872 (15.7100659732879) 1, 3, 7, 15, 39	10.06762 (5.68923324210721) 3, 6, 9, 13, 21	10.0995 (10.8708271648327) 1, 3, 7, 13, 30	12.51246 (13.7006524992782) 1, 4, 8, 16, 39
0.75	6.84628 (4.4210814379314) 2, 4, 6, 9, 15	6.42964 (7.32136336467672) 1, 2, 4, 8, 19	7.41094 (8.80436361429792) 1, 2, 5, 9, 23	7.38726 (3.97723598250358) 3, 5, 7, 9, 15	6.29728 (6.19586252073491) 1, 2, 4, 8, 18	7.57886 (7.89867007034587) 1, 2, 5, 10, 23
1	4.98502 (2.88353288429973) 2, 3, 4, 6, 10	4.09726 (4.13214496174078) 1, 2, 3, 5, 12	4.72018 (5.07003302477385) 1, 2, 3, 6, 14	5.4152 (2.71908014371341) 2, 3, 5, 7, 11	4.03508 (3.4835688738192) 1, 2, 3, 5, 11	4.75252 (4.51330932341458) 1, 2, 3, 6, 14
1.5	2.95216 (1.4376900477688) 1, 2, 3, 4, 6	2.10432 (1.51467594635435) 1, 1, 2, 3, 5	2.26284 (1.8569502146482) 1, 1, 2, 3, 6	3.24794 (1.44958883224025) 1, 2, 3, 4, 6	2.11732 (1.41521591084947) 1, 1, 2, 3, 5	2.28126 (1.72614379582006) 1, 1, 2, 3, 6
2	2.0496 (0.883716843287024) 1, 1, 2, 2, 4	1.41396 (0.722002454393947) 1, 1, 1, 2, 3	1.45056 (0.837167667432318) 1, 1, 1, 2, 3	2.25194 (0.901111800194365) 1, 2, 2, 3, 4	1.43238 (0.714085242778955) 1, 1, 1, 2, 3	1.46042 (0.80724621803334) 1, 1, 1, 2, 3
3	1.30532 (0.480816307217659) 1, 1, 1, 2, 2	1.04196 (0.20571875170751) 1, 1, 1, 1, 1	1.04032 (0.205949377039403) 1, 1, 1, 1, 1	1.40834 (0.521808288625048) 1, 1, 1, 2, 2	1.04754 (0.217165585712452) 1, 1, 1, 1, 1	1.04544 (0.215444040814998) 1, 1, 1, 1, 1
δ = 1.75						
0	8.24972 (5.24631588814551) 3, 5, 7, 10, 18	8.40392 (9.29489626276328) 1, 3, 5, 11, 26	10.09662 (11.4510517904401) 1, 3, 6, 13, 31	8.96292 (4.6796392012698) 3, 6, 8, 11, 18	8.51294 (8.46938174632265) 1, 3, 6, 11, 25	10.80258 (11.3274221053791) 1, 3, 7, 14, 33
0.25	7.7952 (4.89466404513346) 2, 5, 7, 10, 17	7.74552 (8.50914614053377) 1, 2, 5, 10, 24	9.22668 (10.3579574204202) 1, 3, 6, 12, 29	8.4955 (4.40835212084611) 3, 5, 8, 11, 17	7.79284 (7.65484530657108) 1, 3, 5, 10, 22	9.73366 (10.0784222225649) 1, 3, 7, 13, 29
0.5	6.7486 (4.07587662907084) 2, 4, 6, 8, 14	6.25074 (6.67635536253265) 1, 2, 4, 8, 18	7.30778 (8.04917804853049) 1, 2, 5, 9, 22	7.35964 (3.77022325095839) 3, 5, 7, 9, 14	6.22172 (5.83165846105618) 1, 2, 4, 8, 17	7.66814 (7.77909629887589) 1, 2, 5, 10, 23
0.75	5.48332 (3.13914619958632) 2, 3, 5, 7, 11	4.68758 (4.5546227764078) 1, 2, 3, 6, 13	5.37662 (5.62985357541611) 1, 2, 4, 7, 16	6.03132 (3.01987440971127) 2, 4, 5, 8, 12	4.64504 (4.00715417024238) 1, 2, 3, 6, 12	5.61132 (5.49063664791397) 1, 2, 4, 7, 16
1	4.4512 (2.42693559047767) 2, 3, 4, 6, 9	3.47352 (3.11500447425963) 1, 1, 3, 4, 9	3.91056 (3.82850018083749) 1, 1, 3, 5, 11	4.86544 (2.34566912621388) 2, 3, 4, 6, 9	3.48502 (2.85362549455412) 1, 2, 3, 4, 9	4.02486 (3.63178548068516) 1, 2, 3, 5, 11
1.5	2.97244 (1.45020085093416) 1, 2, 3, 4, 6	2.09538 (1.47755416784847) 1, 1, 2, 3, 5	2.25294 (1.80708236318474) 1, 1, 2, 3, 6	3.2791 (1.47164075253782) 1, 2, 3, 4, 6	2.13174 (1.40485730409921) 1, 1, 2, 3, 5	2.30856 (1.75433528159403) 1, 1, 2, 3, 6
2	2.1654 (0.957737534621936) 1, 2, 2, 3, 4	1.49692 (0.813636131003948) 1, 1, 1, 2, 3	1.53706 (0.9371681397073) 1, 1, 1, 2, 3	2.37524 (0.982941639104437) 1, 2, 2, 3, 4	1.52096 (0.799358153689044) 1, 1, 1, 2, 3	1.56394 (0.932142185603186) 1, 1, 1, 2, 3
3	1.4176 (0.543779508553861) 1, 1, 1, 2, 2	1.08248 (0.289272748758786) 1, 1, 1, 1, 2	1.08048 (0.297531074166719) 1, 1, 1, 1, 2	1.5411 (0.581169119233724) 1, 1, 1, 2, 2	1.09174 (0.30371963603354) 1, 1, 1, 1, 2	1.09172 (0.310273052306093) 1, 1, 1, 1, 2
δ = 2						
0	5.94486 (3.27591120335766) 2, 4, 5, 7, 12	5.18828 (5.09959123442411) 1, 2, 4, 7, 15	6.13898 (6.3054531743876) 1, 2, 4, 8, 18	6.53272 (3.13075157649224) 3, 4, 6, 8, 12	5.25102 (4.6711075072735) 1, 2, 4, 7, 14	6.44968 (6.23121211785389) 1, 2, 4, 9, 18
0.25	5.75498 (3.16268007400448) 2, 4, 5, 7, 12	4.9646 (4.69967962522677) 1, 2, 3, 6, 14	5.82902 (6.01784929437441) 1, 2, 4, 8, 17	6.33502 (2.99823304424919) 2, 4, 6, 8, 12	5.0036 (4.36620296305226) 1, 2, 4, 7, 14	6.15664 (6.00869919364583) 1, 2, 4, 8, 18
0.5	5.2747 (2.84241472984326) 2, 3, 5, 7, 11	4.34078 (4.07502897030465) 1, 2, 3, 6, 12	5.07242 (5.03166788423421) 1, 2, 3, 7, 15	5.7989 (2.76443332924356) 2, 4, 5, 7, 11	4.40326 (3.70367867357771) 1, 2, 3, 6, 12	5.34402 (5.04893850974653) 1, 2, 4, 7, 15
0.75	4.63458 (2.45097702342706) 2, 3, 4, 6, 9	3.66372 (3.25898269111139) 1, 2, 3, 5, 10	4.17478 (3.94428993610536) 1, 2, 3, 5, 12	5.12384 (2.4232996310861) 2, 3, 5, 6, 10	3.69632 (2.97102591666582) 1, 2, 3, 5, 9	4.30262 (3.87300156705659) 1, 2, 3, 6, 12
1	3.98338 (2.04248554720571) 1, 3, 4, 5, 8	2.97468 (2.40841751095987) 1, 1, 2, 4, 8	3.3467 (3.05593289921714) 1, 1, 2, 4, 9	4.4007 (2.05689672236662) 2, 3, 4, 6, 8	3.04102 (2.31585073415634) 1, 1, 2, 4, 7.05563	3.44168 (2.95045977465507) 1, 1, 2, 4, 9
1.5	2.9292 (1.40129462731171) 1, 2, 3, 4, 6	2.06762 (1.42405340284762) 1, 1, 2, 3, 5	2.19724 (1.70207941174508) 1, 1, 2, 3, 6	3.24876 (1.45161999380421) 1, 2, 3, 4, 6	2.10362 (1.36377420902504) 1, 1, 2, 3, 5	2.26272 (1.67992697600012) 1, 1, 2, 3, 6
2	2.23148 (0.993013963212312) 1, 2, 2, 3, 4	1.54584 (0.862365101260109) 1, 1, 1, 2, 3	1.6005 (0.99989986798425) 1, 1, 1, 2, 4	2.46638 (1.03028681706528) 1, 2, 2, 3, 4	1.57796 (0.854761282895414) 1, 1, 1, 2, 3	1.63042 (0.993081239278518) 1, 1, 1, 2, 4
3	1.51348 (0.593114934314173) 1, 1, 1, 2, 2	1.12532 (0.356899769051345) 1, 1, 1, 1, 2	1.12864 (0.383372781889635) 1, 1, 1, 1, 2	1.65944 (0.63424516690207) 1, 1, 2, 2, 3	1.13804 (0.371897465108988) 1, 1, 1, 1, 2	1.1395 (0.391053459844264) 1, 1, 1, 1, 2

LP-SR chart Cauchy distribution OOC performance

θ	m=50, n=5			m=100, n=5		
	MW-SR CUSUM chart			MW-SR CUSUM chart		
	K=0	K=3	K=6	K=0	K=3	K=6
δ = 0.5						
0	162.89342 (379.160721749711) 16, 30, 55, 121, 637.049999999996	1881.9241 (1137.44950474591) 110, 722.75, 2159.5, 3001, 3001	2723.88116 (749.688393007913) 508, 3001, 3001, 3001, 3001	144.69976 (272.44031825426) 26, 45, 73, 134, 454	2323.82604 (995.604709848584) 259, 1596, 3001, 3001, 3001	2213.7158 (1034.46460293808) 228, 1298.75, 3001, 3001, 3001
0.25	132.27594 (326.786893358026) 15, 27, 47, 97, 475	1759.20582 (1163.04399732982) 78, 572, 1818, 3001, 3001	1711.7566 (1282.97841016459) 21, 286.75, 1927, 3001, 3001	119.08138 (222.211212574333) 24, 41, 65, 113, 352	2208.75594 (1047.71631738956) 199, 1276, 3001, 3001, 3001	2128.6133 (1074.27832685737) 178.95, 1116, 3001, 3001, 3001
0.5	78.5642 (225.226196644958) 11, 21, 34, 61, 224	***1458.6218 (1188.64304361337) 35, 301, 1121, 3001, 3001	439.12626 (844.01470116031) 2, 13, 61, 322, 3001	74.9224 (135.214322601198) 19, 31, 47, 76, 193	1860.80164 (1148.8034478537) 97, 685, 2114, 3001, 3001	1781.7115 (1145.53672141305) 99, 614.75, 1851.5, 3001, 3001
0.75	45.0327 (148.326241522492) 8, 14, 23, 39, 108	1057.51952 (1136.47844021955) 11, 109, 506, 1989.25, 3001	41.09684 (200.647701835878) 1, 2, 5, 16, 130	43.05516 (68.9146830656954) 12, 21, 31, 48, 101	1325.9624 (1163.55296859939) 31, 249, 901, 2884, 3001	1322.7267 (1135.55804097085) 42, 289, 912, 2681.25, 3001
1	27.02674 (105.966488678765) 5, 10, 15, 25, 60	673.22 (983.157809644806) 4, 30, 166, 839, 3001	3.81122 (24.7490095232324) 1, 1, 2, 3, 10	25.61914 (38.1332632825434) 8, 13, 20, 30, 58	763.6079 (992.874229992058) 8, 61, 270, 1064, 3001	829.7808 (978.924482207666) 15, 110, 370.5, 1190, 3001
1.5	11.85026 (55.8901894092142) 3, 5, 8, 12, 26	218.25404 (592.038723007768) 2, 4, 14, 85, 1452	1.07988 (0.320283057973325) 1, 1, 1, 1, 2	10.56476 (8.35974663788784) 4, 6, 9, 13, 22	142.97646 (436.255396025401) 2, 6, 15, 61, 727	242.6663 (516.519920648907) 2, 14, 54, 198.25, 1214.05
2	6.09406 (24.4413354850754) 2, 3, 5, 7, 13	59.85626 (303.495366863428) 1, 2, 4, 10, 181	1.00082 (0.0286241853487525) 1, 1, 1, 1, 1	5.9043 (2.88394657595404) 3, 4, 5, 7, 11	17.7891 (119.745122318516) 1, 2, 4, 7, 36	56.0917 (216.116820583507) 1, 3, 7, 26, 227.049999999999
3	2.96338 (1.49512664469391) 2, 2, 3, 3, 5	5.27002 (70.0068928642071) 1, 1, 1, 2, 6	1 (0) 1, 1, 1, 1, 1	3.15768 (1.05945243705856) 2, 2, 3, 4, 5	1.92366 (2.39965567766449) 1, 1, 2, 2, 4	4.525 (43.8204495433333) 1, 1, 2, 3, 8
δ = 1						
0	480.30972 (3799.66297928233) 15, 45, 113, 331, 1734.05	357.53472 (551.671202200032) 9, 53, 153, 398.25, 1472.05	357.2254 (544.922275008607) 9, 55, 155, 402, 1455	420.74488 (749.873697555149) 25, 70, 161, 405, 1749	521.5575 (889.658757360282) 16, 93, 251, 604.25, 1851.05	442.9304 (568.90147375844) 16, 91, 235, 551, 1664.05
0.25	415.63916 (2323.2893604249) 13, 37, 90, 265, 1484.09999999999	334.82742 (535.739630954428) 8, 47, 137, 366.25, 1398	339.01772 (529.757865211944) 8, 50, 142, 381, 1380.05	329.2645 (649.262852860682) 21, 54, 119, 301, 1312	465.16554 (801.472773105006) 14, 81, 221, 531, 1666	412.9416 (557.796426014659) 13, 79, 212, 499.25, 1567.44999999999
0.5	256.68086 (1630.74730815823) 9, 23, 49.5, 138, 860.049999999996	279.7128 (493.368354521067) 5, 32, 100, 287, 1224	283.65764 (485.225843532695) 6, 36, 107, 298, 1188	170.09652 (410.969769066912) 15, 32, 61, 139, 624	356.70786 (660.831473220624) 10, 55, 156, 394, 1338	316.8725 (451.268860660926) 9, 57, 156, 384, 1168
0.75	140.80148 (1509.17669836105) 7, 14, 27, 63, 372.049999999996	203.0568 (419.571298893593) 3, 19, 61, 186, 885	214.18634 (418.939789215997) 4, 23, 71, 208, 904.049999999996	71.55984 (209.554424148735) 10, 19, 33, 61, 206	229.86128 (508.79612288878) 6, 31, 88, 235, 889	223.6942 (365.360507329941) 6, 37, 100, 245, 843.149999999998
1	71.4864 (835.579144656674) 5, 10, 17, 33, 144	134.95574 (330.986831379593) 2, 10, 33, 107, 584.049999999996	149.89278 (339.515736811392) 3, 14, 43, 130, 627.049999999996	31.82858 (77.3906967379312) 7, 13, 20, 32, 82	131.07552 (307.154959556432) 4, 15, 44, 126, 522	147.617 (266.10843481729) 4, 22, 61, 160, 565.049999999999
1.5	16.08476 (437.190557912017) 3, 5, 8, 13, 33	51.22026 (187.2424267629) 1, 4, 10, 29, 197	65.02536 (204.009902302989) 1, 5, 14, 45, 258	11.5173 (10.8313455798872) 4, 7, 9, 14, 25	33.95086 (103.964706600196) 2, 5, 11, 28, 124	51.346 (115.945296649677) 2, 7, 18, 49, 203
2	7.55002 (164.176884788906) 2, 4, 5, 8, 15	17.9789 (98.8560716660747) 1, 2, 4, 9, 53	27.2138 (119.038031110301) 1, 2, 5, 16, 95	6.54162 (3.48513854809446) 3, 4, 6, 8, 13	9.1571 (36.0924007627348) 1, 2, 4, 8, 27	16.8511 (40.9105132673691) 1, 3, 6, 16, 62
3	3.27828 (1.68515193271726) 2, 2, 3, 4, 6	3.3539 (28.4638581118999) 1, 1, 2, 3, 7	4.86316 (21.7860342292981) 1, 1, 2, 3, 13	3.5247 (1.321947374481) 2, 3, 3, 4, 6	2.29436 (2.15276679612483) 1, 1, 2, 3, 5	3.0793 (6.47226394162452) 1, 1, 2, 3, 8
δ = 1.25						
0	200.66504 (976.381780570196) 9, 23, 49, 128, 710	171.0948 (379.829655324991) 4, 24, 67, 174, 644.056	177.10356 (359.138337973325) 5, 27, 74, 188, 654	153.84002 (486.296891949498) 14, 32, 60.5, 131, 510	207.52358 (338.377603206877) 7, 38, 103, 244, 740	71.73018 (93.9412990996033) 3, 17, 42, 91, 238
0.25	190.70566 (990.777892749237) 9, 21, 44, 110, 651	162.77412 (390.440481986211) 4, 22, 62, 163, 611	166.3758 (360.088070064428) 5, 25, 68, 174, 629	124.26524 (380.030744328765) 13, 28, 52, 108, 404	192.55688 (322.61882180485) 6, 35, 92, 222, 686.056	50.86098 (65.9616948868315) 2, 12, 30, 64, 170
0.5	141.49304 (1242.29059801246) 7, 16, 31, 72, 409	137.65888 (364.813923337801) 3, 17, 50, 131, 524	142.82232 (347.761001873566) 4, 20, 55, 144, 534	81.98638 (315.303595097863) 10, 21, 36, 70, 247	150.95204 (261.089047203535) 5, 26, 70, 170, 559	24.28566 (31.6185036483364) 1, 6, 14, 30, 80
0.75	82.3285 (631.262948608546) 5, 12, 21, 44, 217.056	102.95774 (275.033848583133) 2, 12, 33, 93, 401	112.35814 (261.201104660914) 3, 14, 40, 108, 436	43.57626 (168.706572790056) 8, 15, 24, 42, 115	108.26252 (206.984771365338) 4, 17, 46, 116, 405	11.08394 (13.1014444636208) 1, 3, 7, 14, 35
1	49.72128 (639.791584674727) 4, 9, 14, 27, 102	73.02882 (214.989982657938) 2, 7, 21, 61, 284	80.72342 (210.592488345208) 2, 9, 27, 75, 310	24.6637 (72.857152055252) 6, 11, 17, 27, 61	66.42528 (142.665903593404) 3, 10, 27, 68, 248	5.67524 (5.92349834971311) 1, 2, 4, 7, 17
1.5	14.83278 (144.599958784341) 3, 5, 8, 13, 31	31.3078 (118.327672367545) 1, 3, 8, 22, 114	40.28272 (158.050197047322) 1, 4, 11, 31, 148	10.92184 (8.16370650724772) 4, 6, 9, 13, 24	22.23658 (52.9713859480586) 2, 4, 9, 21, 79	2.16498 (1.65705054718627) 1, 1, 2, 3, 5
2	7.32542 (123.40792337664) 2, 4, 5, 8, 15	12.79958 (84.1968305457461) 1, 2, 4, 8, 38	18.17158 (81.1069821110029) 1, 2, 5, 13, 63	6.6324 (3.61565646866983) 3, 4, 6, 8, 13	7.76404 (15.9242392127941) 1, 2, 4, 8, 24	1.32806 (0.655259661282819) 1, 1, 1, 2, 3
3	3.40598 (2.04726257848897) 2, 2, 3, 4, 7	2.9516 (9.64235639979869) 1, 1, 2, 3, 7	4.59228 (38.880014371293) 1, 1, 2, 4, 12	3.67786 (1.42223987981125) 2, 3, 3, 4, 6	2.41208 (1.99884716309167) 1, 1, 2, 3, 6	1.01316 (0.114486141264045) 1, 1, 1, 1, 1

δ = 1.5						
0	89.551 (659.880772512489) 6, 14, 27, 59, 265	87.51804 (176.513768122391) 3, 13, 36, 92, 328	93.74388 (187.590106997972) 3, 15, 41, 102.25, 345	56.78376 (282.82129834579) 9, 19, 32, 56, 161	100.80106 (156.935401924754) 4, 19, 51, 120, 358	22.69386 (25.7630784850076) 2, 6, 14, 30, 72
0.25	94.46074 (2790.61443032115) 6, 13, 25, 53, 234	84.07182 (197.500796841893) 3, 12, 34, 87, 314	91.49632 (197.758542053881) 3, 14, 39, 97, 337	51.69996 (165.723225468694) 9, 18, 29, 51, 143	93.98852 (145.399618463016) 4, 18, 47, 110, 342	19.0397 (21.4582499994725) 1, 5, 12, 25, 60
0.5	66.4006 (619.524210216664) 5, 11, 21, 41, 173	73.8514 (175.52633485584) 2, 10, 28, 73, 277	80.87192 (182.94378186771) 2, 12, 33, 82, 298	38.87158 (101.122554179188) 8, 15, 24, 40, 102	77.7733 (130.545793464186) 3, 14, 38, 89, 279	12.56712 (13.9934517271267) 1, 4, 8, 16, 39
0.75	46.27998 (367.543795703362) 5, 9, 16, 30, 113	57.04948 (140.707972220477) 2, 8, 21, 55, 214.056	64.36332 (139.096454573979) 2, 9, 26, 66, 243	27.37824 (50.5203161168773) 6, 12, 19, 30, 69	57.73696 (101.865865838616) 3, 11, 27, 65, 209	7.55372 (7.9048746923957) 1, 2, 5, 10, 23
1	27.95288 (206.119920648489) 4, 7, 12, 22, 66	43.86252 (120.788780678672) 2, 6, 15, 39, 166	49.4378 (118.601467517573) 2, 7, 18, 48, 186.056	18.83544 (27.7183045313836) 5, 9, 14, 22, 45	39.46932 (71.1055031526335) 2, 7, 18, 43, 142	4.74092 (4.55372949159759) 1, 2, 3, 6, 13
1.5	11.94018 (48.8145263085403) 3, 5, 7, 12, 27	20.72276 (65.5885070345979) 1, 3, 7, 17, 75	26.77784 (92.3314565405081) 1, 4, 9, 24, 96	10.14258 (6.98736770248627) 4, 6, 9, 12, 22	16.2801 (31.3569269720007) 1, 4, 8, 17, 56	2.27978 (1.74494815065691) 1, 1, 2, 3, 6
2	6.64198 (13.118876620278) 2, 4, 5, 8, 15	10.08392 (39.7998537434594) 1, 2, 4, 8, 32	13.46916 (44.9901221504923) 1, 2, 5, 12, 48	6.57104 (3.48514508207691) 3, 4, 6, 8, 13	7.09302 (12.9275005196284) 1, 2, 4, 8, 21	1.45418 (0.804856188066955) 1, 1, 1, 2, 3
3	3.67048 (36.3241171317254) 2, 2, 3, 4, 7	3.03748 (15.3983420350487) 1, 1, 2, 3, 7	3.98654 (10.7562220484078) 1, 1, 2, 4, 12	3.7937 (1.5081332167007) 2, 3, 4, 5, 7	2.5298 (2.01734314222124) 1, 1, 2, 3, 6	1.04548 (0.216546778895806) 1, 1, 1, 1, 1
δ = 1.75						
0	43.60644 (449.592891031297) 5, 10, 18, 33, 112	51.25494 (110.223223364116) 2, 8, 22, 54, 188	56.76242 (102.81240423) 2, 10, 26, 63, 206	29.52754 (43.1459063287939) 7, 13, 21, 33, 76	55.85718 (86.2831455040413) 3, 11, 29, 66, 197	10.7649 (11.1383476892125) 1, 3, 7, 14, 33
0.25	38.46414 (170.262935788757) 5, 10, 17, 32, 106	49.21756 (104.139045158197) 2, 8, 21, 51, 178	55.24382 (106.219217575029) 2, 9, 25, 60, 198	27.9874 (39.3764316731141) 7, 13, 20, 32, 71	52.38438 (79.8314465183531) 3, 11, 27, 62, 184	9.86442 (10.3294361902594) 1, 3, 7, 13, 30
0.5	32.44004 (187.212365120981) 4, 9, 15, 27, 86	43.4785 (93.9789317801728) 2, 7, 18, 45, 162	50.1087 (109.045734550942) 2, 8, 22, 53, 181	23.70152 (29.14008326307) 6, 11, 17, 27, 59.0556	45.51888 (74.3871456132997) 2, 9, 23, 52, 160	7.58672 (7.58233140113181) 1, 2, 5, 10, 22
0.75	25.10026 (95.1950076932724) 4, 8, 13, 22, 65	36.72488 (85.9565222624787) 2, 6, 14, 36, 136	42.01908 (89.4540254902323) 2, 7, 18, 44, 153	18.85112 (19.3310907236755) 5, 10, 14, 22, 45	35.31058 (57.1786896327169) 2, 7, 18, 40, 124	5.57702 (5.42517986532534) 1, 2, 4, 7, 16
1	18.76616 (83.0724829266755) 3, 6, 10, 17, 46	28.11452 (70.3735607592132) 1, 4, 11, 27, 104	33.78476 (74.6713376598796) 1, 5, 14, 35, 126	14.85042 (12.8521091404402) 5, 8, 12, 18, 34	26.2975 (44.6339690994869) 2, 6, 13, 29, 91	4.01814 (3.64237234202183) 1, 2, 3, 5, 11
1.5	10.35696 (79.1867702994017) 3, 5, 7, 11, 24	15.34418 (41.5803527960306) 1, 3, 6, 14, 53	19.85812 (51.6471697062712) 1, 3, 8, 19, 70	9.34558 (5.92853585755777) 3, 6, 8, 11, 20	12.7229 (22.2637532569013) 1, 3, 7, 14, 42	2.28652 (1.71891983038991) 1, 1, 2, 3, 6
2	6.42646 (32.7539201967362) 2, 3, 5, 7, 14	8.20088 (24.2495032525155) 1, 2, 4, 7, 26	11.06532 (37.8203212682232) 1, 2, 4, 10, 38	6.42658 (3.32312358581611) 3, 4, 6, 8, 12	6.39872 (9.23841811832451) 1, 2, 4, 7, 19	1.55782 (0.920431307309692) 1, 1, 1, 2, 3
3	3.58298 (2.15152199629425) 2, 2, 3, 4, 7	3.0126 (12.1101830857625) 1, 1, 2, 3, 7	3.84108 (8.59696356798444) 1, 1, 2, 4, 11	3.86922 (1.55955289609064) 2, 3, 4, 5, 7	2.61482 (2.73820494538419) 1, 1, 2, 3, 6	1.09276 (0.314320788926165) 1, 1, 1, 1, 2
δ = 2						
0	23.62894 (73.091606783105) 4, 8, 13, 22, 62	32.73652 (63.4201203130585) 2, 6, 15, 35, 121	38.25988 (68.1040817110437) 2, 7, 18, 42, 138	19.34316 (18.2824245152366) 5, 10, 15, 23, 46	33.8679 (51.7211058567566) 2, 8, 18, 40, 117	6.48418 (6.32665395439052) 1, 2, 4, 9, 19
0.25	23.42558 (237.117913957942) 4, 8, 13, 22, 59	31.75378 (61.1408462467866) 2, 6, 14, 34, 117	36.7561 (69.2916826144061) 2, 7, 17, 40, 132	18.69242 (17.0951273587297) 5, 10, 15, 22, 44	32.98478 (50.4196099855306) 2, 7, 17, 39, 113	6.15396 (5.94552969085733) 1, 2, 4, 8, 18
0.5	19.87402 (72.9214565118937) 4, 7, 12, 20, 51	28.14972 (58.6591711660576) 2, 5, 12, 29, 100	33.91832 (75.2909212510649) 1, 6, 16, 37, 122	16.88918 (15.8724181396771) 5, 9, 13, 20, 39	28.71788 (42.497125178282) 2, 7, 15, 34, 100	5.31598 (4.98083454993802) 1, 2, 4, 7, 15
0.75	16.74356 (52.0960965773058) 3, 6, 10, 17, 42	24.93596 (52.9664498317053) 1, 4, 11, 25, 90	29.23476 (59.9184363290852) 1, 5, 13, 31, 104	14.46838 (18.8695554101846) 5, 8, 12, 17, 32	23.95772 (37.690039309914) 2, 5, 12, 28, 82	4.34612 (3.94381947100021) 1, 2, 3, 6, 12
1	14.35846 (130.403678279466) 3, 6, 9, 14, 33	19.83706 (43.5520516873835) 1, 4, 8, 20, 71	24.18044 (48.4644060871435) 1, 4, 11, 26, 85	12.20232 (9.06013842828943) 4, 7, 10, 15, 27	18.55388 (29.1072458604525) 2, 4, 10, 21, 63	3.49962 (3.01647838355205) 1, 1, 3, 5, 9
1.5	8.7627 (20.0909626858852) 3, 4, 6, 10, 20	12.25744 (32.0568151480204) 1, 3, 5, 12, 41	15.8775 (41.5471233302516) 1, 3, 7, 16, 55	8.58228 (5.06182797452154) 3, 5, 7, 10, 18	10.40818 (15.5957723005266) 1, 3, 6, 12, 33	2.26446 (1.69892278670124) 1, 1, 2, 3, 6
2	6.13134 (29.5350181335702) 2, 3, 5, 7, 13	7.05142 (19.5691081805866) 1, 2, 3, 7, 22	9.32834 (24.1126730427057) 1, 2, 4, 9, 31	6.2432 (3.21971754842282) 3, 4, 6, 8, 12	5.83986 (7.62752245243337) 1, 2, 4, 7, 17	1.63286 (0.988679811746656) 1, 1, 1, 2, 4
3	3.61314 (1.94580447720249) 2, 2, 3, 4, 7	2.91526 (5.35421072463997) 1, 1, 2, 3, 8	3.68912 (8.07240093768502) 1, 1, 2, 4, 11	3.90458 (1.59508178765798) 2, 3, 4, 5, 7	2.6517 (2.13294118502175) 1, 1, 2, 3, 6	1.13678 (0.386929225937403) 1, 1, 1, 1, 2

LP-SR chart Lognormal distribution OOC performance

	m=50, n=5			m=100, n=5		
	MW-SR CUSUM chart			MW-SR CUSUM chart		
θ	**K=0**	**K=3**	**K=6**	**K=0**	**K=3**	**K=6**
	δ = 0.5					
-3	1.99516 (0.0711103053959879) 2, 2, 2, 2, 2	1.00012 (0.0109539034028969) 1, 1, 1, 1, 1	1.00004 (0.00632449207320237) 1, 1, 1, 1, 1	2.01126 (0.105515094032567) 2, 2, 2, 2, 2	1.00028 (0.0167310250296857) 1, 1, 1, 1, 1	1.00008 (0.00894400357244998) 1, 1, 1, 1, 1
-2	2.251 (0.43959397727596) 2, 2, 2, 2, 3	1.17662 (0.385523472913081) 1, 1, 1, 1, 2	1.19934 (0.721036727413553) 1, 1, 1, 1, 2	2.7009 (0.47464059631885) 2, 2, 3, 3, 3	1.29182 (0.456798928310059) 1, 1, 1, 2, 2	1.26218 (0.484279194397986) 1, 1, 1, 1, 2
-1.5	3.0408 (0.651140414716207) 2, 3, 3, 3, 4	1.91538 (0.858856337395356) 1, 1, 2, 2, 3	14.47736 (138.587415362523) 1, 1, 2, 4, 19	3.54872 (0.649919079190779) 3, 3, 3, 4, 5	2.10358 (0.698269958696996) 1, 2, 2, 2, 3	4.59666 (30.4147345536002) 1, 2, 3, 4, 10
-1	4.90402 (1.32484072388302) 3, 4, 5, 6, 7	30.84888 (229.820683215576) 2, 3, 4, 8, 34	761.01542 (1147.93707268852) 2, 13, 89, 1021.25, 3001	5.98122 (1.27651866480598) 4, 5, 6, 7, 8	11.93022 (90.2704129031241) 2, 4, 5, 8, 20	990.40774 (1227.05910751241) 5, 36, 239, 2369, 3001
-0.75	6.763 (2.29521597247998) 4, 5, 6, 8, 11	356.87522 (860.523936493842) 3, 7, 15, 86, 3001	1931.52894 (1286.87116013614) 17, 368, 3001, 3001, 3001	8.34758 (1.98760336962461) 6, 7, 8, 9, 12	343.97676 (821.078399213288) 5, 10, 23, 109, 3001	2516.33802 (972.049869349022) 148, 3001, 3001, 3001, 3001
-0.5	11.33 (40.5381100542112) 6, 7, 9, 12, 20	1496.09334 (1370.6515821108) 8, 62, 1052.5, 3001, 3001	2760.3479 (720.07394325532) 515.95, 3001, 3001, 3001, 3001	12.29382 (4.14394899267173) 8, 10, 11, 14, 20	2104.19202 (1250.78793287966) 27, 573, 3001, 3001, 3001	2972.05814 (248.47033044177) 3001, 3001, 3001, 3001, 3001
-0.25	48.20274 (246.698345267988) 7, 10, 14, 23, 76	2600.9884 (922.735690698337) 122, 3001, 3001, 3001, 3001	2967.27422 (268.75519256465) 3001, 3001, 3001, 3001, 3001	24.61096 (72.0266376044925) 10, 14, 18, 25, 47	2954.52274 (327.675451242414) 3001, 3001, 3001, 3001, 3001	2999.20598 (58.6250701510591) 3001, 3001, 3001, 3001, 3001
0	203.5663 (583.88422837079) 10, 17, 31, 75, 1229.05	2723.88116 (749.688393007913) 508, 3001, 3001, 3001, 3001	2770.20584 (690.379264245722) 670.95, 3001, 3001, 3001, 3001	133.86792 (406.821426756678) 15, 24, 39, 72.25, 422	2919.59366 (405.980898316165) 3001, 3001, 3001, 3001, 3001	2918.98434 (408.239562910773) 3001, 3001, 3001, 3001, 3001
0.25	212.21396 (549.436191221222) 12, 26, 48, 110, 1068	1711.7566 (1282.97841016459) 21, 286.75, 1927, 3001, 3001	1706.44856 (1282.09061150944) 23, 285, 1900.5, 3001, 3001	210.9918 (475.074637558126) 22, 43, 74, 150, 831	1898.03726 (1212.56994346266) 56, 560, 2680.5, 3001, 3001	1872.1674 (1210.95163879268) 60, 549, 2488, 3001, 3001
0.5	56.16194 (221.243705833598) 5, 10, 20, 39, 139	439.12626 (844.01470116031) 2, 13, 61, 322, 3001	442.99896 (837.914135649288) 2, 16, 67.5, 341, 3001	40.86806 (107.357513591508) 7, 14, 24, 42, 105.056	313.88242 (646.151776409619) 3, 18, 63, 244, 1908.05	330.95944 (642.622004210634) 4, 23, 79, 281, 1866
0.75	10.07566 (37.8076230923299) 2, 4, 6, 11, 26	41.09684 (200.647701835878) 1, 2, 5, 16, 130	45.4222 (203.875013985497) 1, 2, 7, 21, 153	8.49148 (6.57768293001833) 3, 5, 7, 10, 20	15.79476 (65.7274310957485) 1, 3, 5, 12, 53	21.27082 (69.3037803933316) 1, 3, 7, 18, 77
1	3.64628 (2.71431566178111) 1, 2, 3, 4, 8	3.81122 (24.7490095232324) 1, 1, 2, 3, 10	4.64372 (21.8889407481527) 1, 1, 2, 4, 14	3.65036 (1.73006697356913) 2, 2, 3, 4, 7	2.4907 (2.58244204003455) 1, 1, 2, 3, 6	3.09484 (5.00728537558393) 1, 1, 2, 3, 9
1.5	1.5202 (0.601031766867356) 1, 1, 1, 2, 2	1.07988 (0.320283057973325) 1, 1, 1, 1, 2	1.08448 (0.382395154416322) 1, 1, 1, 1, 2	1.754 (0.531497553891827) 1, 1, 2, 2, 2	1.0609 (0.249624590633064) 1, 1, 1, 1, 2	1.0593 (0.255782756448456) 1, 1, 1, 1, 2
2	1.05422 (0.226806570505372) 1, 1, 1, 1, 2	1.00082 (0.0286241853487525) 1, 1, 1, 1, 1	1.00098 (0.0319227188879585) 1, 1, 1, 1, 1	1.1213 (0.326478853478805) 1, 1, 1, 1, 2	1.0007 (0.026448515846448) 1, 1, 1, 1, 1	1.00028 (0.0167310250296857) 1, 1, 1, 1, 1
3	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1	1 (0) 1, 1, 1, 1, 1
	δ = 1					
-3	***2.04412 (0.218664999228587) 2, 2, 2, 2, 2	1.05814 (0.235118791288405) 1, 1, 1, 1, 2	1.05386 (0.233923480659587) 1, 1, 1, 1, 2	***2.23534 (0.425063169392456) 2, 2, 2, 2, 3	1.09084 (0.288426347955876) 1, 1, 1, 1, 2	1.07868 (0.277940645858162) 1, 1, 1, 1, 2
-2	***2.79302 (0.69367780948944) 2, 2, 3, 3, 4	***1.66326 (0.787171242596745) 1, 1, 2, 2, 3	1.98352 (1.56333530950523) 1, 1, 1, 2, 5	3.22916 (0.686276266457086) 2, 3, 3, 4, 4	1.80544 (0.782175582828336) 1, 1, 2, 2, 3	2.1069 (1.38391860115178) 1, 1, 2, 3, 5
-1.5	***3.99414 (1.26458595751846) 2, 3, 4, 5, 6	3.1255 (2.40048015948891) 1, 2, 3, 4, 7	6.52706 (11.2504097343512) 1, 2, 4, 7, 21	4.64418 (1.26375791561572) 3, 4, 4, 5, 7	3.32094 (2.03047284463483) 1, 2, 3, 4, 7	6.92272 (8.80676438787706) 1, 2, 4, 8, 21
-1	***7.6725 (3.77713238907265) 4, 5, 7, 9, 14	19.10614 (46.6963463803723) 2, 4, 8, 18, 65	72.83972 (375.75013372397) 2, 8, 22, 58, 251	8.91224 (3.36066720519621) 5, 7, 8, 11, 15	18.38244 (31.281760670036) 2, 5, 10, 21, 59	94.24224 (249.725911151399) 3, 13, 36, 91, 342
-0.75	***14.1825 (18.3211611817849) 5, 8, 11, 16, 31	116.46336 (526.630845866881) 3, 11, 30, 85, 419	333.4495 (1349.6476108167) 4, 25, 78, 238, 1241.05	15.45948 (7.87236162858086) 7, 10, 14, 19, 30	121.74718 (298.570357515972) 4, 17, 45, 116, 452.056	541.33248 (1678.93566998761) 9, 53, 161, 461, 2055.05
-0.5	***45.54928 (105.784869713276) 8, 14, 23, 42, 134	511.49662 (1608.79117520251) 8, 48, 153, 457, 2013.05	947.69286 (2766.73429117756) 13, 90, 284, 833, 3676	41.6839 (439.486665817495) 11, 19, 29, 45, 99	845.80852 (1696.55002691205) 17, 107, 328, 887.25, 3281	1788.37378 (3907.5009497455) 35, 229, 683, 1863, 6799.15
-0.25	187.29186 (553.517800469731) 14, 32, 67, 165, 672	698.48834 (1450.91807525072) 16, 99, 282, 736, 2634.05	889.5001 (2026.94246353344) 19, 116, 336, 884, 3395	***196.83726 (374.127096208461) 23, 50, 95, 202, 674	1095.65936 (1778.24061359479) 33, 196, 529, 1283, 3952.05	1274.2818 (2277.6748157628) 36, 216.75, 584, 1425, 4632.05
0	244.81386 (623.685357564538) 13, 39, 92, 232, 892	282.11938 (617.509063380678) 6, 36, 106, 282, 1094.05	309.69104 (748.908241064811) 6, 38, 111, 302, 1178.05	276.07406 (500.769470651288) 22, 61, 134, 304, 966	308.60434 (538.758434928112) 9, 53, 142, 348, 1135	302.90104 (555.773816167704) 9, 54, 144, 344, 1089
0.25	86.77924 (289.733340872601) 6, 15, 32, 75, 314	71.84988 (186.616146610386) 2, 11, 29, 72, 263	72.64718 (163.55075320837) 2, 11, 30, 75, 270	69.29992 (127.450292426241) 8, 20, 37, 72, 226	66.0725 (101.202762826585) 3, 13, 34, 78, 233	69.30274 (106.331756767874) 3, 14, 37, 83, 237
0.5	19.54142 (34.6830918852523) 3, 7, 12, 21, 57	19.17144 (33.776995515216) 1, 4, 9, 21, 67	20.7814 (36.3090713322886) 1, 4, 10, 23, 72	17.3612 (16.4002095687061) 4, 8, 13, 21, 43	17.72972 (24.1726571784879) 1, 4, 10, 22, 58	19.88348 (25.9855399800612) 1, 5, 12, 25, 65
0.75	7.46312 (7.01911285167182) 2, 4, 6, 9, 18	6.80158 (9.53403731170322) 1, 2, 4, 8, 22	7.53634 (10.2793692760331) 1, 2, 4, 9, 25	7.66686 (4.84846655066918) 2, 4, 7, 10, 17	6.33446 (6.77878717378787) 1, 2, 4, 8, 19	7.37942 (8.31955556287085) 1, 2, 5, 9, 23
1	4.16214 (2.6039712428759) 1, 2, 4, 5, 9	3.20946 (3.1779660944966) 1, 1, 2, 4, 9	3.56244 (3.8963784315307) 1, 1, 2, 4, 10	4.45692 (2.30996338330646) 2, 3, 4, 6, 9	3.07956 (2.58064398956307) 1, 1, 2, 4, 8	3.47702 (3.28386473456991) 1, 1, 2, 4, 10
1.5	2.09238 (0.956527173939501) 1, 1, 2, 3, 4	1.44604 (0.800937668250763) 1, 1, 1, 2, 3	1.47554 (0.904509851334646) 1, 1, 1, 2, 3	2.28478 (0.934953386194567) 1, 2, 2, 3, 4	1.44742 (0.744584737775785) 1, 1, 1, 2, 3	1.47686 (0.855849981003563) 1, 1, 1, 2, 3
2	1.4292 (0.556123678214576) 1, 1, 1, 2, 2	1.0846 (0.298805330763214) 1, 1, 1, 1, 2	1.08332 (0.309805902451751) 1, 1, 1, 1, 2	1.55512 (0.584780834990767) 1, 1, 2, 2, 2	1.08418 (0.290956046007665) 1, 1, 1, 1, 2	1.0842 (0.302179063212071) 1, 1, 1, 1, 2
3	1.03374 (0.18056097154504) 1, 1, 1, 1, 1	1.0006 (0.024487792735154) 1, 1, 1, 1, 1	1.00072 (0.0268234224125073) 1, 1, 1, 1, 1	1.0551 (0.228177631025975) 1, 1, 1, 1, 2	1.00068 (0.0260682026811175) 1, 1, 1, 1, 1	1.00052 (0.0227978067980211) 1, 1, 1, 1, 1

δ = 1.25						
-3	2.16286 (0.381863243571224) 2, 2, 2, 2, 3	***1.15994 (0.380791408046359) 1, 1, 1, 1, 2	***1.1682 (0.430897288746946) 1, 1, 1, 1, 2	2.43916 (0.517053035265074) 2, 2, 2, 3, 3	1.22198 (0.433576566909222) 1, 1, 1, 1, 2	1.21982 (0.476344104271391) 1, 1, 1, 1, 2
-2	3.14918 (0.906411473468416) 2, 3, 3, 4, 5	2.02254 (1.13970958018196) 1, 1, 2, 2, 4	2.55406 (2.20794361719233) 1, 1, 2, 3, 7	3.6291 (0.928218951437879) 2, 3, 3, 4, 5	2.16596 (1.11482829854768) 1, 1, 2, 3, 4	2.76456 (2.14173288409058) 1, 1, 2, 3, 7
-1.5	4.63628 (1.70227075288108) 3, 3, 4, 5, 8	4.03824 (3.51501135254158) 1, 2, 3, 5, 10	7.186 (9.22113358465713) 1, 2, 4, 9, 23	5.36626 (1.71491470080836) 3, 4, 5, 6, 8	4.21392 (3.0925118441471) 1, 2, 3, 5, 10	8.16448 (9.18367862725295) 1, 3, 5, 10, 25
-1	9.01392 (4.83458723142669) 4, 6, 8, 11, 18	18.21466 (30.1184203348926) 2, 4, 10, 21, 61	38.67728 (78.3936270561381) 2, 7, 18, 42, 135	10.29102 (4.4071709545599) 5, 7, 9, 12, 19	19.53204 (25.6091704283954) 2, 6, 12, 24, 63	52.22708 (85.052075764119) 2, 11, 27, 61, 184
-0.75	15.40672 (11.6999810526655) 5, 9, 12, 19, 35	54.70576 (99.3180039274878) 3, 10, 25, 60, 198	100.63302 (206.880522371465) 3, 16, 44, 108, 369	17.02922 (9.20539086492364) 7, 11, 15, 21, 34	67.96656 (104.632038683363) 4, 14, 35, 80, 237	154.0756 (255.154618461486) 5, 28, 74, 178, 559
-0.5	31.87812 (44.4457500078407) 7, 14, 22, 36, 87	122.92364 (191.959781257176) 5, 23, 61, 144, 435	181.1316 (305.274866139219) 6, 32, 89, 211, 646	32.59614 (24.1110975279202) 10, 18, 26, 40, 75	185.12664 (254.581592339379) 7, 39, 101, 230, 646	291.41678 (413.825525730043) 10, 60, 156, 359, 1011.05
-0.25	56.42854 (80.1489194590558) 9, 19, 34, 64, 171	132.19594 (196.626556262294) 5, 26, 69, 159, 466	156.09078 (239.739086416009) 6, 31, 81, 187, 546	59.14328 (57.6915969379315) 13, 26, 43, 71, 159	176.0436 (230.495321993548) 8, 41, 103, 224, 582	207.8839 (276.245739107334) 9, 47, 119, 263, 695
0	52.04678 (75.4257869207757) 7, 16, 31, 59, 163	68.90828 (108.798387346612) 3, 14, 35, 81, 244	74.2521 (115.779526357489) 3, 15, 38, 88, 259.056	52.7948 (55.2197743333812) 10, 22, 37, 63, 149	76.18328 (100.706009167513) 4, 18, 44, 96, 255	81.19486 (105.652670478867) 4, 19, 48, 102, 273
0.25	27.0277 (38.8642264977965) 4, 10, 17, 30, 80	27.32914 (41.2049468366758) 2, 6, 15, 33, 94	29.01714 (42.3294455652284) 2, 6, 16, 35, 99	25.8086 (23.3534107503781) 6, 12, 20, 32, 66	27.31976 (33.1998885268364) 2, 7, 17, 35, 88	29.8485 (36.1352933146869) 2, 8, 18, 38, 96
0.5	12.22412 (14.9538624637801) 3, 5, 9, 15, 32	11.42328 (15.1494905585265) 1, 3, 7, 14, 37	12.36698 (15.7947160512845) 1, 3, 7, 15, 40	12.45482 (8.80307836221927) 3, 7, 10, 16, 29	11.34138 (12.602941576407) 1, 3, 7, 15, 35	12.64066 (14.1128621555645) 1, 4, 8, 16, 39
0.75	6.65282 (4.88159837017067) 2, 4, 5, 8, 15	5.60946 (6.36168750645288) 1, 2, 4, 7, 17	6.17332 (7.03713227118917) 1, 2, 4, 8, 19	7.09044 (4.21421591742948) 2, 4, 6, 9, 15	5.5218 (5.43729308048783) 1, 2, 4, 7, 16	6.23336 (6.35895524725852) 1, 2, 4, 8, 18
1	4.27896 (2.58639422886422) 1, 2, 4, 5, 9	3.23306 (2.98152994732847) 1, 1, 2, 4, 9	3.51362 (3.51654685986642) 1, 1, 2, 4, 10	4.65176 (2.44935274898855) 2, 3, 4, 6, 9	3.218 (2.64353092758819) 1, 1, 2, 4, 8	3.56672 (3.22340134763342) 1, 1, 3, 5, 10
1.5	2.35152 (1.12344956824) 1, 2, 2, 3, 4	1.64144 (1.01430533011813) 1, 1, 1, 2, 4	1.68648 (1.14258099031377) 1, 1, 1, 2, 4	2.56134 (1.1352811024868) 1, 2, 2, 3, 5	1.64746 (0.955067427711693) 1, 1, 1, 2, 3	1.6985 (1.09920967755852) 1, 1, 1, 2, 4
2	1.6271 (0.665893728975702) 1, 1, 2, 2, 3	1.1808 (0.449260945006278) 1, 1, 1, 1, 2	1.18406 (0.484506564797864) 1, 1, 1, 1, 2	1.76608 (0.704053511763807) 1, 1, 2, 2, 3	1.18826 (0.446656649388822) 1, 1, 1, 1, 2	1.19162 (0.475506359290761) 1, 1, 1, 1, 2
3	1.11936 (0.32520040821947) 1, 1, 1, 1, 2	1.00772 (0.087980431978527) 1, 1, 1, 1, 1	1.0075 (0.08833971970819) 1, 1, 1, 1, 1	1.15962 (0.368489038213385) 1, 1, 1, 1, 2	1.0076 (0.0868469391965943) 1, 1, 1, 1, 1	1.00748 (0.0866267838719278) 1, 1, 1, 1, 1
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-2	3.53502 (1.15183337944456) 2, 3, 3, 4, 6	2.43538 (1.60041103529988) 1, 1, 2, 3, 5	3.20932 (2.90800864302109) 1, 1, 2, 4, 9	4.0791 (1.19510324049226) 3, 3, 4, 5, 6	2.58794 (1.52324422265743) 1, 2, 2, 3, 5	3.50202 (3.00707445360693) 1, 1, 3, 4, 9
-1.5	5.23664 (2.12211488327837) 3, 4, 5, 6, 9	4.87014 (4.45179882189829) 1, 2, 4, 6, 13	7.8329 (9.22574440775364) 1, 2, 5, 10, 24	6.01972 (2.13065762515668) 3, 4, 6, 7, 10	5.15518 (4.18543301518379) 1, 2, 4, 7, 13	9.2334 (10.1172333969437) 1, 3, 6, 12, 28
-1	9.5632 (5.1883392465492) 4, 6, 8, 12, 19	16.85802 (22.4409842394462) 2, 5, 10, 20, 55	28.00898 (39.2479959674803) 2, 6, 15, 34, 94	10.93328 (4.84359965838972) 5, 7, 10, 13, 20	18.58986 (21.2612813824125) 2, 6, 12, 24, 58	38.12426 (49.8001267117345) 2, 9, 22, 48, 127
-0.75	14.2601 (9.46274795511205) 5, 8, 12, 18, 31	34.43002 (47.3577989144551) 2, 8, 19, 42, 119	53.1943 (73.7161881070758) 2, 11, 29, 66, 183	15.8604 (8.09685019125711) 6, 10, 14, 20, 31	42.50466 (53.3560815251) 3, 11, 25, 54, 142	77.1329 (98.2135888549579) 4, 18, 45, 99, 259
-0.5	21.02766 (17.2602205440332) 6, 11, 16, 26, 51	53.52432 (71.2634210380133) 3, 12, 30, 67, 181	71.4585 (93.6193489050196) 3, 16, 41, 91, 240	22.96452 (14.1504539106178) 8, 14, 20, 28, 49	70.83828 (85.2743222831024) 4, 17, 42, 92, 236	102.47004 (122.664020118513) 5, 26, 63, 133, 335
-0.25	26.13776 (24.8137320169364) 6, 12, 19, 32, 68	50.19828 (65.1307245873045) 3, 12, 29, 63, 169	59.16864 (74.7456985982443) 3, 14, 35, 76, 198	27.57966 (18.9996482603516) 8, 15, 23, 34, 63	60.20842 (70.3651917236621) 3, 15, 37, 79, 193	73.71626 (86.244221562534) 4, 19, 46, 97, 237
0	***22.2746 (21.1358238377638) 5, 10, 16, 27, 59	30.41988 (40.1857357609982) 2, 7, 17, 38, 102	33.9252 (43.3398967683708) 2, 8, 20, 43, 113	***23.50224 (17.1508459853373) 6, 12, 19, 30, 55	***33.0286 (37.8329043111016) 2, 9, 21, 44, 104	***37.161 (42.3261119050047) 2, 10, 24, 49, 118
0.25	14.74644 (13.3506281542047) 3, 7, 11, 18, 37	15.60028 (19.0552157150351) 1, 4, 9, 20, 50	17.31112 (21.5084777852347) 1, 4, 10, 22, 56	15.64274 (10.624202688487) 4, 8, 13, 20, 35	16.30008 (17.969035859461) 1, 5, 11, 21, 50	18.42598 (20.126480629858) 1, 5, 12, 24, 58
0.5	9.20688 (6.92600606820418) 2, 5, 7, 12, 22	8.4711 (9.54991250256127) 1, 3, 5, 11, 26	9.32654 (10.6048479896893) 1, 3, 6, 12, 29	9.87202 (6.06144998685289) 3, 6, 9, 13, 21	8.57286 (8.72149830654425) 1, 3, 6, 11, 25	9.73942 (10.0925574191037) 1, 3, 7, 13, 29
0.75	6.05376 (3.97094261229051) 2, 3, 5, 8, 13	4.95576 (4.97991755092153) 1, 2, 3, 6, 14	5.51212 (5.78496001894986) 1, 2, 4, 7, 16	6.6149 (3.72053152615278) 2, 4, 6, 8, 14	5.03316 (4.64573482577745) 1, 2, 4, 7, 14	5.6221 (5.44303629630397) 1, 2, 4, 7, 16
1	4.27072 (2.49774207153363) 1, 3, 4, 5, 9	3.23436 (2.86031449641125) 1, 1, 2, 4, 9	3.53928 (3.33202627984573) 1, 1, 2, 5, 10	4.7142 (2.47526986391747) 2, 3, 4, 6, 9	3.2637 (2.64109481420799) 1, 1, 2, 4, 8	3.6084 (3.20509826574754) 1, 1, 3, 5, 10
1.5	2.54796 (1.25346370582687) 1, 2, 2, 3, 5	1.7959 (1.17404887369102) 1, 1, 1, 2, 4	1.86416 (1.34351166532781) 1, 1, 1, 2, 4	2.7991 (1.28676816305273) 1, 2, 3, 3, 5	1.81244 (1.13752675825341) 1, 1, 1, 2, 4	1.89274 (1.30418913911526) 1, 1, 1, 2, 4
2	1.81744 (0.77645598978827) 1, 1, 2, 2, 3	1.28954 (0.589706319680287) 1, 1, 1, 1, 2	1.30852 (0.649879878498427) 1, 1, 1, 1, 3	1.97192 (0.82490309909859) 1, 1, 2, 2, 3	1.30386 (0.588299262553036) 1, 1, 1, 1, 2	1.31954 (0.643461319155538) 1, 1, 1, 1, 3
3	1.2334 (0.431518440119635) 1, 1, 1, 1, 2	1.02722 (0.165286472502621) 1, 1, 1, 1, 1	1.02712 (0.168952882489557) 1, 1, 1, 1, 1	1.2933 (0.471040919232026) 1, 1, 1, 2, 2	1.0299 (0.173224103784094) 1, 1, 1, 1, 1	1.02848 (0.17038036914329) 1, 1, 1, 1, 1

δ = 1.75						
-3	2.54042 (0.64808535232057) 2, 2, 2, 3, 4	1.47052 (0.682686055815568) 1, 1, 1, 2, 3	1.56458 (0.92790443674598) 1, 1, 1, 2, 3	2.88712 (0.700705305644611) 2, 2, 3, 3, 4	1.55296 (0.704105925058918) 1, 1, 1, 2, 3	1.66284 (0.965599182909576) 1, 1, 1, 2, 4
-2	3.9238 (1.40232410314371) 2, 3, 4, 5, 7	2.87798 (2.06209024145969) 1, 1, 2, 4, 7	3.87346 (3.60960222364795) 1, 1, 3, 5, 11	4.51094 (1.44359343155887) 3, 3, 4, 5, 7	3.03718 (1.97587340918253) 1, 2, 3, 4, 7	4.27644 (3.87645733432335) 1, 2, 3, 6, 12
-1.5	5.70288 (2.4399177792654) 3, 4, 5, 7, 10	5.58392 (5.26776161380224) 1, 2, 4, 7, 16	8.60684 (9.68115797365141) 1, 3, 5, 11, 27	6.53304 (2.44308569881505) 3, 5, 6, 8, 11	5.90312 (5.07399735658026) 1, 3, 4, 8, 16	10.08118 (10.7591981585293) 1, 3, 7, 13, 30
-1	9.42982 (5.10503633635647) 4, 6, 8, 12, 19	15.06832 (17.9035276720445) 1, 4, 9, 19, 48	23.28144 (28.90134352032) 1, 6, 14, 30, 77	10.59634 (4.66624410860401) 5, 7, 10, 13, 19	16.5774 (18.1448734900158) 2, 5, 11, 21, 51	30.17292 (35.808549589701) 2, 8, 19, 39, 96
-0.75	12.19816 (7.45592143398042) 4, 7, 10, 15, 26	23.83602 (28.996854776188) 2, 6, 14, 31, 78	34.42776 (41.5230815897435) 2, 8, 21, 45, 113	13.61744 (6.62724726070524) 6, 9, 12, 17, 26	27.80938 (31.6152917833932) 2, 8, 17, 36, 88	47.01462 (54.1653443074636) 3, 12, 29, 62, 152
-0.5	14.99484 (10.2926173607029) 5, 8, 12, 19, 34	29.43808 (35.1098946168683) 2, 8, 18, 38, 96	39.06678 (46.5918342245927) 2, 10, 24, 51, 127	16.47384 (8.93299454954678) 6, 10, 15, 20, 33	35.4337 (39.6996862155321) 3, 10, 23, 47, 112	51.30774 (56.9926261834036) 3, 14, 33, 68, 161
-0.25	15.7481 (11.6913754603641) 4, 8, 13, 20, 37	25.75776 (30.6218274064216) 2, 7, 16, 33, 84	31.42 (36.9355996942042) 2, 8, 19, 41, 102	17.19602 (10.0299435768584) 6, 10, 15, 22, 36	29.53196 (32.8843343576744) 2, 8, 19, 39, 93	38.10432 (42.2908520564549) 2, 10, 25, 51, 119
0	13.77042 (10.3112152264798) 3, 7, 11, 17, 33	17.79394 (20.9876880322102) 1, 5, 11, 23, 57	20.5253 (23.8425856250488) 1, 5, 13, 27, 66	14.88578 (8.91728008695942) 5, 9, 13, 19, 32	18.94596 (20.3439140082518) 2, 6, 13, 25, 58	22.91772 (24.6566946092411) 2, 7, 15, 31, 71
0.25	10.4419 (7.56652162059877) 3, 6, 9, 13, 24	10.91772 (11.9666709667259) 1, 3, 7, 14, 34	12.48264 (14.0333259534277) 1, 3, 8, 16, 39	11.34396 (6.71591196993169) 3, 7, 10, 14, 24	11.4638 (11.7894507668079) 1, 4, 8, 15, 34	13.30242 (13.9119615078917) 1, 4, 9, 18, 40
0.5	7.59586 (5.06951722163928) 2, 4, 6, 10, 17	6.87044 (7.13238751228735) 1, 2, 5, 9, 20	7.73484 (8.19483698931624) 1, 2, 5, 10, 23	8.37024 (4.79833959028084) 3, 5, 7, 11, 17	7.04344 (6.70749825024618) 1, 2, 5, 9, 20	8.12624 (8.05002108812085) 1, 3, 6, 11, 24
0.75	5.58764 (3.45833463273418) 2, 3, 5, 7, 12	4.54056 (4.27072355321594) 1, 2, 3, 6, 13	5.03032 (4.9445737605777) 1, 2, 3, 7, 15	6.16006 (3.36893867432398) 2, 4, 6, 8, 12	4.63338 (4.12488424916401) 1, 2, 3, 6, 13	5.22814 (4.90621377021743) 1, 2, 4, 7, 15
1	4.21186 (2.40379926479274) 1, 3, 4, 5, 9	3.20096 (2.68872454208185) 1, 1, 2, 4, 8	3.52154 (3.21174443791013) 1, 1, 2, 5, 10	4.66064 (2.42484481727136) 2, 3, 4, 6, 9	3.27272 (2.63558773863716) 1, 1, 2, 4, 8	3.62238 (3.15150468431121) 1, 1, 3, 5, 10
1.5	2.7093 (1.33996620113658) 1, 2, 2, 3, 5	1.92256 (1.28707271648635) 1, 1, 2, 2, 4	2.02438 (1.50062342145292) 1, 1, 1, 2, 5	3.00444 (1.4163828610966) 1, 2, 3, 4, 6	1.96906 (1.26941519791552) 1, 1, 2, 2, 4	2.07476 (1.50229693480404) 1, 1, 2, 3, 5
2	1.98792 (0.871153977081569) 1, 1, 2, 2, 4	1.40714 (0.713657625576951) 1, 1, 1, 2, 3	1.43436 (0.803158945468351) 1, 1, 1, 2, 3	2.15874 (0.934408408984795) 1, 2, 2, 3, 4	1.42286 (0.716421444119111) 1, 1, 1, 2, 3	1.44804 (0.796035697601959) 1, 1, 1, 2, 3
3	1.35216 (0.506585103382778) 1, 1, 1, 2, 2	1.06436 (0.254202837870972) 1, 1, 1, 1, 2	1.06354 (0.260353651943816) 1, 1, 1, 1, 2	1.42398 (0.545735206980751) 1, 1, 1, 2, 2	1.0692 (0.263007116738063) 1, 1, 1, 1, 2	1.06706 (0.266766526550079) 1, 1, 1, 1, 2
δ = 2						
-3	2.74906 (0.77360266650375) 2, 2, 3, 3, 4	1.66252 (0.861186438839842) 1, 1, 1, 2, 3	1.83748 (1.23756934397941) 1, 1, 1, 2, 4	3.1229 (0.818760647185701) 2, 3, 3, 4, 5	1.75112 (0.872533077813923) 1, 1, 2, 2, 3	1.93692 (1.26284314507233) 1, 1, 2, 2, 4
-2	4.26556 (1.61575063032697) 2, 3, 4, 5, 7	3.31302 (2.55649157882123) 1, 2, 3, 4, 8	4.51404 (4.32180245244143) 1, 2, 3, 6, 13	4.87374 (1.634641200214) 3, 4, 5, 6, 8	3.43648 (2.36915121238585) 1, 2, 3, 4, 8	5.01932 (4.63384248626599) 1, 2, 4, 7, 14
-1.5	5.99262 (2.63171124055833) 3, 4, 5, 7, 11	6.0904 (5.79224816826566) 1, 2, 4, 8, 17	9.08744 (9.79630305765762) 1, 3, 6, 12, 28	6.78338 (2.59717359485531) 3, 5, 6, 8, 12	6.37194 (5.54568081734917) 1, 3, 5, 8, 17	10.72892 (11.1889203890451) 1, 3, 7, 14, 32
-1	8.8027 (4.59879067663595) 3, 6, 8, 11, 17	12.99668 (14.5697012495621) 1, 4, 8, 17, 41	19.53502 (22.3908079480221) 1, 5, 12, 26, 62	9.86092 (4.31184283031883) 4, 7, 9, 12, 18	14.25106 (14.8995083403237) 2, 5, 9, 18, 43	24.87022 (27.4271737907653) 2, 7, 16, 33, 78
-0.75	10.4387 (5.98786768400806) 4, 6, 9, 13, 21	17.46096 (20.1664515865779) 2, 5, 11, 22, 56	24.63044 (28.1882662995357) 2, 6, 15, 32, 78	11.56282 (5.41445472545477) 5, 8, 11, 14, 22	19.47556 (21.1636509300455) 2, 6, 13, 25, 60	31.70604 (34.8788336699948) 2, 9, 21, 42, 99
-0.5	11.47594 (7.09061257241806) 4, 7, 10, 14, 25	18.79002 (21.4690835116578) 2, 5, 12, 24, 60	25.36132 (28.6678671126698) 2, 7, 16, 34, 81	12.71934 (6.37971974156229) 5, 8, 11, 16, 25	21.37072 (23.5051393638313) 2, 6, 14, 28, 67	31.77864 (34.4009899321599) 2, 9, 21, 43, 99
-0.25	11.41046 (7.45744023487263) 3, 6, 10, 14, 25	16.50586 (18.6883929427069) 1, 5, 10, 21, 52	20.61508 (23.2537414478493) 1, 6, 13, 27, 65	12.51834 (6.63674502840748) 4, 8, 11, 16, 25	18.04746 (19.2973048705747) 2, 5, 12, 24, 55.0556	24.25554 (25.8319527930836) 2, 7, 16, 32, 75
0	10.10974 (6.59337292921701) 3, 6, 9, 13, 22	12.14342 (13.486433493751) 1, 4, 8, 16, 38	14.58234 (15.915297234363) 1, 4, 9, 19, 45	11.20834 (6.09310733235016) 4, 7, 10, 14, 23	12.93562 (13.2754419874744) 1, 4, 9, 17, 39	16.20998 (16.8889855573466) 1, 5, 11, 22, 49
0.25	8.29208 (5.38885424451259) 2, 5, 7, 11, 18	***8.41814 (8.84076139942949) 1, 3, 6, 11, 25	9.87598 (10.5381431043995) 1, 3, 6, 13, 30	9.12792 (5.00724853830509) 3, 6, 8, 12, 18	***8.76446 (8.41075952134613) 1, 3, 6, 12, 25	***10.59598 (10.6625766890763) 1, 3, 7, 14, 31
0.5	6.56786 (4.04571407497459) 2, 4, 6, 8, 14	5.86456 (5.79048586915917) 1, 2, 4, 8, 17	6.66672 (6.72899621261601) 1, 2, 5, 9, 20	7.25744 (3.90242863168367) 2, 4, 7, 9, 15	6.04222 (5.54361002450949) 1, 2, 4, 8, 17	7.09158 (6.85749032861674) 1, 2, 5, 10, 21
0.75	5.16256 (3.03609594775463) 2, 3, 5, 7, 11	4.21778 (3.8129519592637) 1, 2, 3, 5, 12	4.74702 (4.53573726949121) 1, 2, 3, 6, 14	5.72088 (3.01121128011096) 2, 4, 5, 7, 11	4.32424 (3.71362144571641) 1, 2, 3, 6, 12	4.96496 (4.54664113795148) 1, 2, 4, 7, 14
1	4.12906 (2.31786776297082) 1, 2, 4, 5, 8	3.16978 (2.62249734059847) 1, 1, 2, 4, 8	3.48124 (3.12080804460272) 1, 1, 2, 4, 10	4.59222 (2.35079688542416) 2, 3, 4, 6, 9	3.23804 (2.55269804006382) 1, 1, 2, 4, 8	3.61366 (3.13285775297156) 1, 1, 3, 5, 10
1.5	2.84016 (1.42485500267065) 1, 2, 3, 4, 5	2.03022 (1.39564526585883) 1, 1, 2, 3, 5	2.15972 (1.62397729899323) 1, 1, 2, 3, 5	3.1373 (1.49225106676622) 1, 2, 3, 4, 6	2.08386 (1.3654980014983) 1, 1, 2, 3, 5	2.19236 (1.6117845970823) 1, 1, 2, 3, 5
2	2.12362 (0.949987444718562) 1, 1, 2, 3, 4	1.49966 (0.816733264575718) 1, 1, 1, 2, 3	1.54266 (0.917364134648347) 1, 1, 1, 2, 3	2.32182 (1.02439878731747) 1, 2, 2, 3, 4	1.52676 (0.822178461131425) 1, 1, 1, 2, 3	1.57276 (0.931946003192812) 1, 1, 1, 2, 3
3	1.4643 (0.56640261849547) 1, 1, 1, 2, 2	1.11056 (0.336895764851871) 1, 1, 1, 1, 2	1.11082 (0.3492583101765) 1, 1, 1, 1, 2	1.5546 (0.610496760127261) 1, 1, 1, 2, 3	1.11568 (0.344819540644541) 1, 1, 1, 1, 2	1.11508 (0.354625307588412) 1, 1, 1, 1, 2

Appendix C: Implementation codes

GetARL algorithm

```
#PARAMETERS
OOC.sd=1
OOC.mean=0
m=50
n=5
K=3
h=12
replicate=50000
  rep = 1
  N = m + n
  RL = rep(0,times = replicate)
  for(rep in 1:replicate){
    x.m = rlnorm(m, meanlog=0, sdlog=1)
    i = 0
    IC = 0
    Cj = 0
    while (IC == 0) {
      i = i + 1
      yj = c((rlnorm(n, meanlog=OOC.mean, sdlog=OOC.sd)))
      x <- c(x.m,yj)
      #WRS statistic
      r = rank(x)
      t.WRS = sum(r[1:m])
      avg.WRS= m*(N+1)/2
      sd.WRS = sqrt(m*n*(N+1)/12)
      WRS = (t.WRS-m.WRS)/s.WRS
      #Mood statistic
      a.mood = (r - (N+1)/2)^2
      t.mood = sum(a.mood[1:m])
      avg.mood = (m*((N^2-1)))/12
      sd.mood = sqrt((m*n*(N+1)*((N^2-4)))/(180))
      mood = abs((t.mood-m.mood)/s.mood)
      #Lepage statistic
      LP = MW^2 + mood^2
      #CUSUM plotting statistic
      Cj <- max(0,Cj+(LP-2)-K)
      if (Cj > h){
        IC = 1
        RL[rep] = i
      }
    }
  }
  ARL = mean(RL)
  SDRL = sd(RL)
  QRL = quantile(RL, probs = c (0.05, 0.25, 0.5, 0.75, 0.95))
  return(ARL)
```

GetH algorithm

```
control = 0
while (control == 0){
  h = (h.min + h.max)/2
  ARL = getARL(m, n, K, h, replicate)
  if ((ARL < (A0 + tolerance)) & (ARL > (A0 - tolerance))){
    control = 1
  }
  else if(ARL > A0){
    h.max = (h.min + h.max)/2
  }
  else{
    h.min = (h.min + h.max)/2
  }
}
return(h)
```

Curriculum Vitae

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Recently graduated Industrial Engineer from Barcelona Tech, I've been living in Monterrey for the past 6 months as a Master's exchange student. Very pro-active person, eager to learn many fields. I love travelling, sports and reading.

WORK EXPERIENCE& EDUCATION

ITESM - ETSEIB — 2015 – present

MSc in Industrial Engineering - Thesis

Master thesis performed in ITESM in order to become an Industrial Engineer for Barcelona Tech (ETSEIB faculty)

- Creation of a Control Chart for the Quality Management industry
- Been working on nonparametric statistics under the MSc in Manufacturing Systems in Tec, coming from Barcelona, where I studied Industrial Engineering with an specialization in Electronics

Worldsensing S.L. — Feb/Jul – 2015

Corporate Strategy intern

Internship done in an Internet of Things start-up as an assistant for the CEO of the company.

- Providing support on the creation of an Enterprise Asset Management solution
- Studying of fields of interest for the CEO: G3-PLC, LoRa & Digital Oilfields
- Analysis and setting of internal processes to enhance patent & international projects activity

BEST Barcelona — 2012 – 2015

European BEST Engineering Competition coordination

Member for 4 years of the European engineering students association, BEST.

- Coordination of the biggest engineering competition in Europe (6500 participants from over 30 countries)
- Getting sponsorships and funds for the association
- Forming, leading and motivating the new members

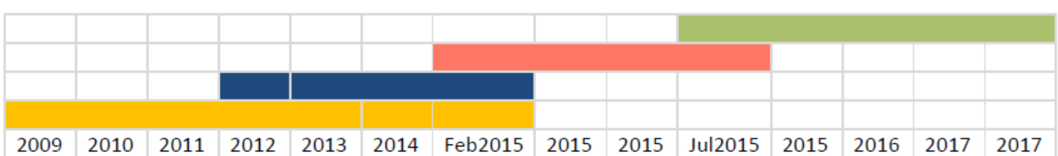
ETSEIB — 2009 – 2015

BSc in Industrial Engineering

Student in Barcelona Tech (ETSEIB), as a prerequisite to become an Industrial Engineer according to Spanish law

- Studied a wide arrange of fields: industrial organization, mechanics, chemistry, electronics, statistics, heat transfer, system dynamics...

TIMELINE



SKILLS

Public Speaking	Leadership	Team Work
Sales	Communication	Work under pressure
Initiative	Strategy	English/Spanish

This document was typed in using Microsoft Word by Victor Aguilar